Math 581: Partial Differential Equations II

Assignment 4

Due: March 05, 2010, Friday

1. Find all eigenvalues and the corresponding eigenfunctions to the following Neumann boundary problem
$$\begin{cases} -u_{xx} = \lambda u \\ u_x(0) = u_x(1) = 0. \end{cases}$$

2. Find all eigenvalues and the corresponding eigenfunctions to the following periodic boundary problem
$$\begin{cases} -u_{xx} = \lambda u \\ u(x) = u(x+2L) \\ \int_0^{2L} u(x)dx = 0. \end{cases}$$

3. Consider the eigenvalue problem to the fourth-order elliptic equation
$$\begin{cases} \Delta^2 u = \lambda u \quad \text{in } \Omega \\ u|_{\partial\Omega} = 0, \end{cases}$$
where $\Delta^2 u = \Delta(\Delta u)$.
   (a) Prove that
      i. The eigenvalues are
      $$0 < \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_k \leq \cdots .$$
      ii. Let $w_k \in H_0^2(\Omega)$ be the corresponding normalized eigenfunctions. Then $\{w_k\}_{k=1}^{\infty}$ is the orthonormal basis of $L^2(\Omega)$.

   (b) Find all eigenvalues λ_k and eigenfunctions w_k to
   $$\begin{cases} u_{xxxx} = \lambda u \\ u(0) = u(1) = 0, \end{cases}$$
   and verify that $\{w_k\}_{k=1}^{\infty}$ is the orthonormal basis of $L^2(\Omega)$.