

MATH 251: Linear Algebra I

Midterm Examination

Instructor: Dr. Ming Mei

- [25 pts] Let V be the vector space of functions $f : \mathbf{R} \rightarrow \mathbf{R}$ with the usual addition and scalar multiplication. Test if the following subsets of V are subspaces. If yes, prove it; if no, give the reason.
 - The set of all odd functions: $W_1 = \{f(t) \mid f(-t) = -f(t)\}$;
 - The set of all bounded functions with up and low bounds 2 and -2:
 $W_2 = \{f(t) \mid -2 \leq f(t) \leq 2\}$.
- [25 pts] Let $S = \{u_1, u_2, u_3\}$, where $u_1 = (1, 2, 3)$, $u_2 = (4, 5, 6)$ and $u_3 = (7, 8, 9)$. Find a basis and the dimension of the spanning space $\text{span } S$.
- [25 pts] Let V be a 5-dimensional vector space, and U and W be a 1-dimensional and 4-dimensional subspaces of V , respectively. Show all possibilities of $U + W$.
- [25 pts] Let v_1, v_2, v_3 and v_4 be linearly independent vectors in a vector space V . Show that $w_1 = v_1 + v_2 + v_3 + v_4$, $w_2 = v_2 + v_4$, $w_3 = v_1 - v_2 + v_4$ and $w_4 = 2v_1 + v_2 + 3v_3 + v_4$ are linearly independent.