TEXTBOOK - MATH 203 (APR 04)

- *Single Variable Calculus - Early Transcendentals*, 5th ed. by J. Stewart

Concordia UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	203/4	All
Examination	**Date**	**Pages**
Final	April 2004	3

Instructors	Course Examiner
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Special Instructions

▷ Calculators are **not** allowed.

MARKS

[10] **1.** (a) Suppose $f(x) = \sqrt{1-x^2}$ and $g(x) = \cos x$. Find $f \circ g \circ f$ and $g \circ f \circ g$. Simplify.

(b) Find the inverse of the function $f(x) = \sqrt{3^x - 1}$. Determine the domain and range of f and f^{-1}.

[10] **2.** Evaluate the limits:

(a) $\lim\limits_{x \to 3} \dfrac{\sqrt{2x-5}-1}{9-x^2}$ (b) $\lim\limits_{x \to \infty} \dfrac{(2x-1)^2\,(x^2+1)}{(x+1)^3\,(2-3x)}$

Do not use l'Hopital's rule.

[12] **3.** (a) Consider the function $f(x) = \dfrac{|x+1|}{x^2-2x-3}$.

Calculate both one-sided limits at the point(s) where the function is undefined.

(b) Find parameters a and b such that the function

$$f(x) = \begin{cases} x+1, & \text{if} \quad x \le 0 \\ ax^2+b, & \text{if} \quad 0 < x \le 1 \\ x-1, & \text{if} \quad x > 1 \end{cases}$$

will be continuous at every point. Sketch the graph of this function.

[15] **4.** Find derivatives of the functions (do not simplify the answer):

(a) $f(x) = \dfrac{2\sqrt{x} - x^2 + 3x}{x\sqrt{x}}$;

(b) $f(x) = \left(x^3 - \dfrac{1}{x}\right)\sqrt{\cos x}$;

(c) $f(x) = \ln\left(1 + \sin^2(3x)\right)$;

(d) $f(x) = \dfrac{\arctan(x^2)}{x^2 + 1}$;

(e) $f(x) = x^{\sin x}$ (use logarithmic differentiation).

[12] **5.** Given the function $f(x) = \dfrac{1}{\sqrt{2x}}$,

(a) Use appropriate differentiation rules to find the derivative of the function.

(b) Use the definition of derivative to verify (a).

(c) Find the differential of the function.

(d) Use the differential above (with appropriate choice of x_0 and Δx) to approximate $\dfrac{1}{\sqrt{4.1}}$.

[15] **6.** (a) The equation of a curve defined implictly is $2e^{x+y} = x^2y^2 - \dfrac{x}{y}$.

Verify that the point $(1, -1)$ belongs to the curve. Find an equation of the tangent line to the curve at this point.

(b) Let $f(x) = \dfrac{3x^2 + 5}{x^2}$. Find $f^{(3)}(x)$; $f^{(33)}(x)$.

(c) Use l'Hopital's rule to evaluate $\lim\limits_{x \to 0} \dfrac{e^{x^2} - 1}{x \sin x}$.

[10] **7.** (a) A particle is moving along the plane curve $xy^3 = 8$. At the moment when $x = 1$ the x-coordinate is increasing at a rate of 3 cm/sec. Is the y-coordinate increasing or decreasing at this moment? How fast?

(b) Find the point(s) x at which the tangent line to the graph of function $f(x) = x^5 - 10x^2 + 10$ has minimum slope (do not graph).

[16] **8.** Given the function $f(x) = \dfrac{x}{x^2+4}$,

(a) Find the domain and check for symmetry. Find asymptotes (if any).

(b) Calculate $f'(x)$ and use it to determine interval(s) where the function is increasing, interval(s) where the function is decreasing, and local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine interval(s) where the function is concave upward, interval(s) where the function is concave downward and inflection point(s) (if any).

(d) Sketch the graph of the function.

[5] **Bonus Question**

Given the equation $x^5 - 3x^3 - 1 = 0$,

(a) Use the Intermediate Value Theorem to show that there is a root between -1 and 0.

(b) Use the Mean Value Theorem to show that the equation has exactly one root in this interval.