

Review for Test #3

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1. Simplify

$$\frac{2 \cos^2 x - \cos x - 1}{\cos x - 1}$$

2. Use trigonometric identities to evaluate

① $\sin 75^\circ$, ② $\cos \frac{\pi}{12}$

3. Prove

$$\frac{1 + \sin \alpha}{1 + \csc \alpha} = \frac{\tan \alpha}{\sec \alpha}$$

4. Find $\tan^{-1} \sqrt{3}$ exactly in radians.

5. Solve: $2 \sin^2 x = \sqrt{2} \sin x$ in $[0, 2\pi)$

Solutions to Review Questions.

$$\begin{aligned}
1. \quad & \frac{2 \cos^2 x - \cos x - 1}{\cos x - 1} \\
&= \frac{2 \cos^2 x - 1 - \cos x + 1 - 1}{\cos x - 1} \\
&= \frac{2 \cos^2 x - 2 - [\cos x - 1]}{\cos x - 1} \\
&= \frac{2 [\cos^2 x - 1]}{\cos x - 1} - \frac{\cos x - 1}{\cos x - 1} \\
&= \frac{2 (\cancel{\cos x - 1}) (\cos x + 1)}{\cancel{\cos x - 1}} - 1 \\
&= 2 (\cos x + 1) - 1 \\
&= 2 \cos x + 2 - 1 \\
&= 2 \cos x + 1
\end{aligned}$$

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(3)

$$\begin{aligned} 2. \quad (1) \quad \sin 75^\circ &= \sin (30^\circ + 45^\circ) \\ &= \sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2} + \sqrt{6}}{4} \quad // \end{aligned}$$

$$\begin{aligned} (2) \quad \cos \frac{\pi}{12} &= \cos 15^\circ = \cos \frac{30^\circ}{2} \\ &= \sqrt{\frac{1 + \cos 30^\circ}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} \\ &= \sqrt{\frac{\frac{2 + \sqrt{3}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{3}}{4}} \\ &= \frac{\sqrt{2 + \sqrt{3}}}{2} \quad // \end{aligned}$$

$$\begin{aligned} 3. \quad \text{LHS} &= \frac{1 + \sin^2 \alpha}{1 + \csc \alpha} = \frac{1 + \sin^2 \alpha}{1 + \frac{1}{\sin \alpha}} \\ &= \frac{1 + \sin^2 \alpha}{\frac{\sin \alpha + 1}{\sin \alpha}} = \frac{1 + \sin^2 \alpha}{1} \cdot \frac{\sin \alpha}{\sin \alpha + 1} \\ &= \sin \alpha \end{aligned}$$

(4)

$$\begin{aligned} \text{RHS} &= \frac{\tan \alpha}{\sec \alpha} = \frac{\frac{\sin \alpha}{\cos \alpha}}{\frac{1}{\cos \alpha}} \\ &= \frac{\sin \alpha}{\cancel{\cos \alpha}} \cdot \frac{\cancel{\cos \alpha}}{1} \\ &= \sin \alpha \end{aligned}$$

So, $\text{RHS} = \text{LHS}$

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4. Let $\theta = \tan^{-1} \sqrt{3}$

then $\tan \theta = \sqrt{3}$ and

$\theta \in [0, \frac{\pi}{2})$, because $\sqrt{3} > 0$, & $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$

since $\tan 60^\circ = \sqrt{3}$, $60^\circ = \frac{\pi}{3}$

So, $\theta = \frac{\pi}{3}$ //

5. $2 \sin^2 x = \sqrt{2} \sin x$ $x \in [0, 2\pi)$

$$\sin^2 x = \frac{\sqrt{2}}{2} \sin x$$

$$\sin^2 x - \frac{\sqrt{2}}{2} \sin x = 0$$

$$\sin x \left(\sin x - \frac{\sqrt{2}}{2} \right) = 0$$

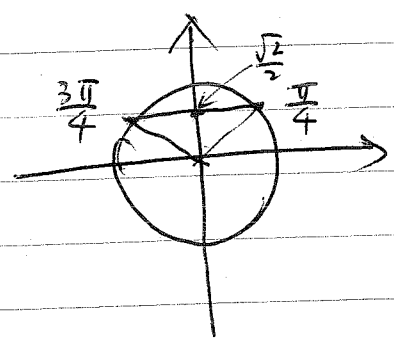
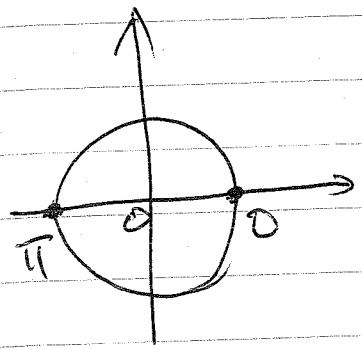
$$\sin x = 0 \quad \text{or} \quad \sin x - \frac{\sqrt{2}}{2} = 0$$

(5)

then the solutions are:

$$\sin x = 0 \Rightarrow \boxed{x = 0}, \text{ and } \boxed{x = \pi}$$

$$\sin x = \frac{\sqrt{2}}{2} \Rightarrow \boxed{x = \frac{\pi}{4}} \text{ and } \boxed{x = \frac{3\pi}{4}}$$



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