Review for Test #3

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1. Simplify

$$\frac{2\cos^2 x - \cos x - 1}{\cos x - 1}$$

2. Use trigonometric identities to evaluate

① $\sin 75°$, ② $\cos \frac{\pi}{12}$

3. Prove

$$\frac{1+\sin\alpha}{1+\csc\alpha} = \frac{\tan\alpha}{\sec\alpha}$$

4 Find $\tan^{-1}\sqrt{3}$ exactly in radians.

5. Solve: $2\sin^2 x = \sqrt{2}\sin x$ in $[0, 2\pi)$

Solutions to Review Questions.

1. $\dfrac{2\cos^2 x - \cos x - 1}{\cos x - 1}$

$= \dfrac{2\cos^2 x - 1 - \cos x + 1 - 1}{\cos x - 1}$

$= \dfrac{2\cos^2 x - 2 - [\cos x - 1]}{\cos x - 1}$

$= \dfrac{2[\cos^2 x - 1]}{\cos x - 1} - \dfrac{\cos x - 1}{\cos x - 1}$

$= \dfrac{2\cancel{(\cos x - 1)}(\cos x + 1)}{\cancel{\cos x - 1}} - 1$

$= 2(\cos x + 1) - 1$

$= 2\cos x + 2 - 1$

$= 2\cos x + 1$

//

2. ① $\sin 75^\circ = \sin(30^\circ + 45^\circ)$

$= \sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$ //

② $\cos \frac{\pi}{12} = \cos 15^\circ = \cos \frac{30^\circ}{2}$

$= \sqrt{\frac{1+\cos 30^\circ}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}}$

$= \frac{\sqrt{2+\sqrt{3}}}{2}$ //

3. $LHS = \frac{1+\sin\alpha}{1+\csc\alpha} = \frac{1+\sin\alpha}{1+\frac{1}{\sin\alpha}}$

$= \frac{1+\sin\alpha}{\frac{\sin\alpha+1}{\sin\alpha}} = \frac{\cancel{1+\sin\alpha}}{1} \cdot \frac{\sin\alpha}{\cancel{\sin\alpha+1}}$

$= \sin\alpha$

$$RHS = \frac{\tan\alpha}{\sec\alpha} = \frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{1}{\cos\alpha}}$$

$$= \frac{\sin\alpha}{\cancel{\cos\alpha}} \cdot \frac{\cancel{\cos\alpha}}{1}$$

$$= \sin\alpha$$

So. $RHS = LHS$ //

4. Let $\theta = \tan^{-1}\sqrt{3}$.

then $\tan\theta = \sqrt{3}$ and

$\theta \in [0, \frac{\pi}{2})$, because $\sqrt{3} > 0$, & $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$

since $\tan 60° = \sqrt{3}$. $60° = \frac{\pi}{3}$

So. $\theta = \frac{\pi}{3}$. //

5. $2\sin^2 x = \sqrt{2}\sin x$ $\quad x \in [0, 2\pi)$

$\sin^2 x = \frac{\sqrt{2}}{2}\sin x$

$\sin^2 x - \frac{\sqrt{2}}{2}\sin x = 0$

$\sin x(\sin x - \frac{\sqrt{2}}{2}) = 0$

$\sin x = 0$ or $\sin x - \frac{\sqrt{2}}{2} = 0$

then the solutions are:

$\sin x = 0 \Rightarrow \boxed{x = 0}$, and $\boxed{x = \pi}$

$\sin x = \frac{\sqrt{2}}{2} \Rightarrow \boxed{x = \frac{\pi}{4}}$ and $\boxed{x = \frac{3\pi}{4}}$