CHAMPLAIN
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MATHEMATICS 201-009

Review Questions for Test #1

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/10 1. Consider the numbers:

$$-5,\ \frac{2}{3},\ \sqrt{2},\ 0,\ -3.4,\ \sqrt{3},\ 8,\ 2\tfrac{1}{2},\ \frac{0}{0},\ \pi,\ 1.\bar{2},\ 1.23057\ldots$$

(a) Which are natural numbers?

(b) Which are integers?

(c) Which are rational numbers?

(d) Which are irrational numbers?

(e) Which are real numbers?

/10 2. Simplify:

$$\frac{-2^3a^4b^{-6}}{4^2a^{-2}b^4}$$

/10 3. Rationalize the denominator:

$$\frac{\sqrt{3} - \sqrt{5}}{\sqrt{3} + \sqrt{5}}$$

/10 4. Simplify: $\dfrac{\frac{8}{y} - \frac{y}{8}}{(y+8)^2}$. Also give restrictions for y.

/10 5. Perform the operation and simplify the answer.

$$\frac{x^2 + 13x + 40}{x^2 - x - 6} \div \frac{x^2 + 14x + 24}{x^2 - 9}$$

/10 6. Perform the operation and simplify the answer.

$$\frac{x}{x^2 + 9x + 20} - \frac{5}{x^2 + 11x + 30}$$

/10 7. Given the points A(5, -4) and B(-8, -10)

(a) Find the equation of line $\overline{AB}$

(b) Find the equation of a circle having $\overline{AB}$ as its diameter

/10 8. Graph the function

$$y = f(x) = \begin{cases} x + 2 & \text{for } x < -2 \\ x^2 - 3 & \text{for } -2 \le x \le 2 \\ \sqrt{x} & \text{for } x > 2 \end{cases}$$

Also determine the domain, range the x and y intercepts of f(x).

/10 9. Given $f(x) = 1 + 2x$ $g(x) = \sqrt{x}$

Find the following new functions and determine their domains.

(a) $(f + g)(x)$

(b) $(\frac{f}{g})(x)$

(c) $(f \circ g)(x)$

(d) $(g \circ f)(x)$

(e) $(f \circ f)(-2)$

/10 10. For given line: $y= 2x +1$, find the equation to

(a) a line which is parallel to the given line and passes through the point (0,0).

(b) a line which is perpendicular to the given line and passes through the point (0,0).

Solutions to Review Questions for Test #1

by Dr. M. Me

1. a) natural numbers: 8

   b) integers: $-5, 0, 8,$

   c) rational numbers: $-5, \frac{2}{3}, 0, -3.4, 8, 2\frac{1}{}, 1.$

   d) irrational numbers: $\sqrt{2}, \sqrt{3}, \pi, 1.23057\ldots$

   e) real numbers:

   $-5, \frac{2}{3}, \sqrt{2}, 0, -3.4, \sqrt{3}, 8, 2\frac{1}{2}, \pi, 1.\bar{2}, 1.2$ //

2. $$\frac{-2^3 a^4 b^{-6}}{4^2 a^{-2} b^4} = -\frac{2^3}{2^{2\cdot 2}} a^{4-(-2)} b^{-6-4}$$

   $$= -2^{3-4} a^6 b^{-10}$$

   $$= -\frac{a^6}{2 b^{10}}$$ //

3. $$\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}+\sqrt{5}} = \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}+\sqrt{5}} \cdot \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}$$

   $$= \frac{(\sqrt{3}-\sqrt{5})^2}{(\sqrt{3})^2-(\sqrt{5})^2}$$

   $$= \frac{(\sqrt{3}-\sqrt{5})^2}{3-5} = \frac{(\sqrt{3}-\sqrt{5})^2}{-2}$$ //

4. $$\frac{\frac{8}{y}-\frac{y}{8}}{(y+8)^2} = \frac{\frac{8^2-y^2}{8y}}{(y+8)^2}$$

$$= \frac{\cancel{(8+y)}(8-y)}{8y\,\cancel{(y+8)}}$$

$$= \frac{8-y}{8y}$$

For y: since denominators cannot be zero, we then have $y \neq 0$ for $\frac{8}{y}$, and $y+8 \neq 0$ for $\frac{\frac{8}{y}-\frac{y}{8}}{(y+8)^2}$

So, we restrict y to be,

$y \neq 0$ and $y \neq -8$. //

5 $$\frac{x^2+13x+40}{x^2-x-6} \div \frac{x^2+11x+24}{x^2-9}$$

$$= \frac{(x+5)(x+8)}{(x-3)(x+2)} \div \frac{\cancel{(x+3)}(x+8)}{(x-3)\cancel{(x+3)}}$$

$$= \frac{(x+5)(x+8)}{(x-3)(x+2)} \div \frac{x+8}{x-3}$$

$$= \frac{(x+5)\cancel{(x+8)}}{\cancel{(x-3)}(x+2)} \times \frac{\cancel{x-3}}{\cancel{x+8}}$$

$$= \frac{x+5}{x+2} \quad //$$

6. $$\frac{x}{x^2+9x+20} - \frac{5}{x^2+11x+30}$$

$$= \frac{x}{(x+4)(x+5)} - \frac{5}{(x+5)(x+6)}$$

$$= \frac{x(x+6) - 5(x+4)}{(x+4)(x+5)(x+6)}$$

$$= \frac{x^2+6x-5x-20}{(x+4)(x+5)(x+6)}$$

$$= \frac{x^2+x-20}{(x+4)(x+5)(x+6)}$$

$$= \frac{(x-4)\cancel{(x+5)}}{(x+4)\cancel{(x+5)}(x+6)}$$

$$= \frac{x-4}{(x+4)(x+6)}$$

//

7. a) By the point-point formula:

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

We have: $(x_1, y_1) = (5, -4)$, $(x_2, y_2) = (-8, -10)$, and

$$\frac{y-(-4)}{x-5} = \frac{-10-(-4)}{-8-5}$$

i.e.

$$\frac{y+4}{x-5} = \frac{-6}{-13} = \frac{6}{13}$$

So

$$y+4 = \frac{6}{13}(x-5)$$

$$y+4 = \frac{6}{13}x - \frac{30}{13}$$

$$y = \frac{6}{13}x - \frac{30}{13} - \frac{52}{13}$$

$$\boxed{y = \frac{6}{13}x - \frac{82}{13}}$$

b) Since $\overline{AB}$ is the diameter, its middle point is just the centre of the circle. By the middle point formula, the middle point of $\overline{AB}$ is:

$$\bar{x} = \frac{x_1+x_2}{2} = \frac{5+(-8)}{2} = -\frac{3}{2}$$

$$\bar{y} = \frac{y_1+y_2}{2} = \frac{-4+(-10)}{2} = -7$$

$$|\overline{AB}| = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} = \sqrt{[5-(-8)]^2+[-4-(-10)]^2} = \sqrt{205}$$

Therefore, the equation for the circle is:

$$\sqrt{(x-\bar{x})^2+(y-\bar{y})^2} = \left(\frac{\overline{AB}}{2}\right),$$

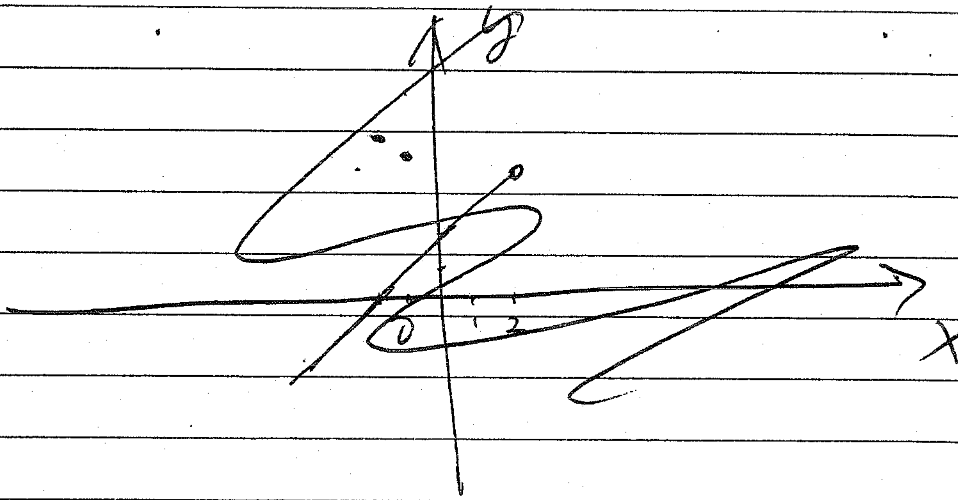
$$(x-\bar{x})^2+(y-\bar{y})^2 = \left(\frac{\overline{AB}}{2}\right)^2,$$

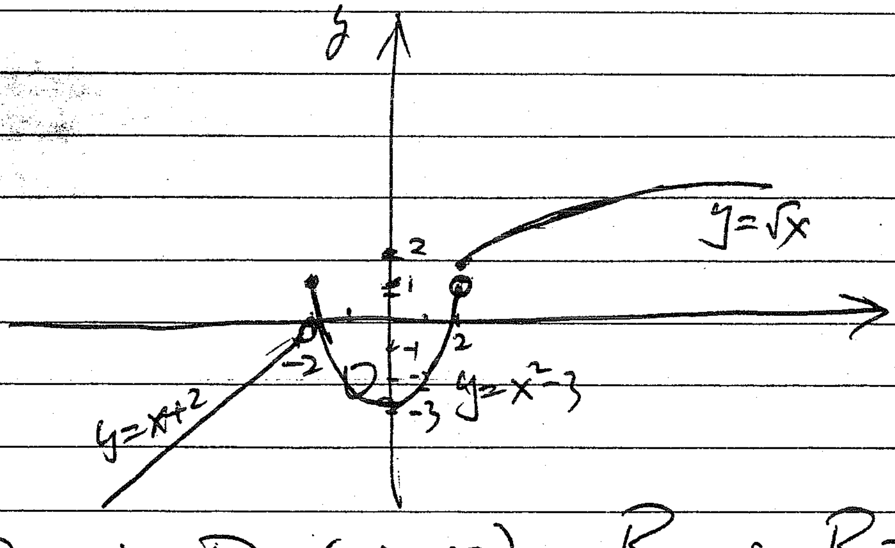
$$\left(x+\frac{3}{2}\right)^2+(y+7)^2 = \left(\frac{\sqrt{205}}{2}\right)^2 = \frac{205}{4}$$

i.e.

$$\left(x+\frac{3}{2}\right)^2+(y+7)^2 = \frac{205}{4}$$

8.





Domain: $D=(-\infty,\infty)$. Range: $R=(-\infty,\infty)$

9. $f(x) = 1+2x, \quad g(x) = \sqrt{x}$

a) $(f+g)(x) = f(x) + g(x)$

$= 1+2x+\sqrt{x}$

b) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{1+2x}{\sqrt{x}}$

c) $(f \circ g)(x) = f(g(x))$

$= 1+2\,g(x)$

$\therefore = 1+2\sqrt{x}$

d) $(g \circ f)(x) = g(f(x))$

$= \sqrt{f(x)}$

$= \sqrt{1+2x}$

e) $(f \circ f)(x) = f(f(x))$

$= 1+2f(x)$

$= 1+2(1+2x)$

$= 1+2+4x$

$= 3+4x$

$(f \circ f)(-2) = 3+4(-2) = 3-8 = -5.$

10. a) $l_1: y = 2x+1$, $m_1 = 2$

$l_2: y = m_2 x + b_2$ $m_2 = 2$ ($\because l_1 \parallel l_2$)

and $(0,0)$ is on l_2, i.e.

$$0 = 2\cdot 0 + b_2 \;;\; \Rightarrow b_2 = 0$$

So, l_2 is:

$$\boxed{y = 2x}$$

b) $l_1: y = 2x+1$, $m_1 = 2$

$l_2: y = m_2 x + b_2$

$\because l_1 \perp l_2$ $\therefore m_1 \cdot m_2 = -1$

$\Rightarrow m_2 = \frac{-1}{m_1} = -\frac{1}{2}$

So, l_2 is $y = -\frac{1}{2}x + b_2$

On the other hand, $(0,0)$ is on l_2:

$$0 = -\frac{1}{2}\cdot 0 + b_2 \Rightarrow b_2 = 0$$

So, l_2 is:

$$\boxed{y = -\frac{1}{2}x}$$