CHAMPLAIN COLLEGE ST.-LAMBERT

MATH 201-NYB: Calculus II

Review Questions for Test # 2

Instructor: Dr. Ming Mei

1. Let R_1 be a region bounded by the curves $y = x^2 - 1$ and $y = -2x^2 + 2$. Find its area.

2. Let R_2 be a region bounded by the $y = \sqrt{x}$ and $y = x$, V_2 be the solid obtained by rotating R_2 about the x-axis. Find the volume of V_2.

3. Let R_2 be the region specified in Q.2, and V_3 be the solid obtained by rotating R_2 about the y-axis. Find the volume of V_3.

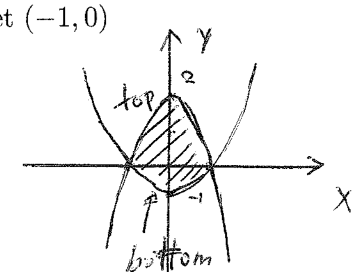
4. Let $f(x) = 1 + 6x^{3/2}$, find the arc length of $f(x)$ for x from 0 to 1.

5. Evaluate the following integrals, and test them to be convergent or not.

$$\text{a).} \quad \int_0^\infty xe^{-x^2}dx \qquad \text{b).} \quad \int_0^1 \frac{1}{x^2+x}dx.$$

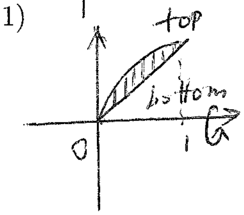
Solutions

Solution to Q1. Solving the equations $y = x^2 - 1$ and $y = -2x^2 + 2$, we get $(-1, 0)$ and $(1, 0)$, as their intersection points. So, the area of R_1 is

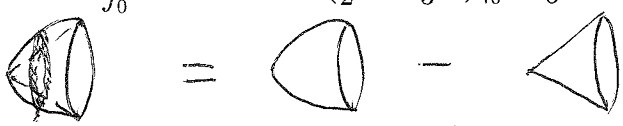


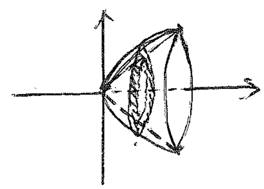
$$\begin{aligned} A &= \int_{-1}^{1} [Top \; - \; Bottom] \; dx = \int_{-1}^{1} [(-2x^2 + 2) - (x^2 - 1)] \; dx \\ &= \int_{-1}^{1} [3 - 3x^2] dx = [3x - x^3] \Big|_{-1}^{1} = 4. \end{aligned}$$

Solution to Q2. Solving the equations $y = \sqrt{x}$ and $y = -x$, we get $(0, 0)$ and $(1, 1)$ as their intersection points. The volume of V_2 is

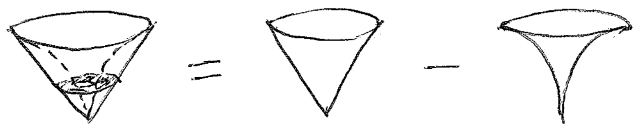


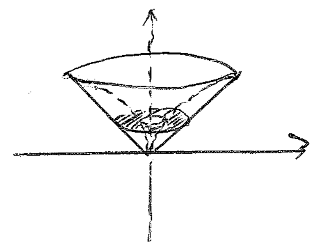
$$\begin{aligned} V_2 &= \pi \int_0^1 [Top]^2 \; dx \; - \pi \int_0^1 [Bottom]^2 \; dx = \pi \int_0^1 [\sqrt{x}]^2 dx - \pi \int_0^1 [x]^2 \; dx \\ &= \pi \int_0^1 [x - x^2] dx = \pi \Big(\frac{1}{2}x^2 - \frac{1}{3}x^3\Big) \Big|_0^1 = \frac{\pi}{6}. \end{aligned}$$





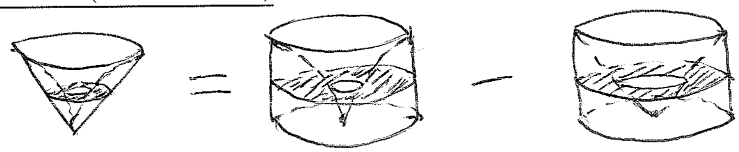
Solution to Q3. Method 1(Disc-Method).





$$\begin{aligned} V_3 &= \pi \int_0^1 [Right]^2 \; dy \; - \pi \int_0^1 [Left]^2 \; dy = \pi \int_0^1 [y]^2 dy - \pi \int_0^1 [y^2]^2 \; dy \\ &= \pi \int_0^1 [y^2 - y^4] dy = \pi \Big(\frac{1}{3}y^2 - \frac{1}{5}y^5\Big) \Big|_0^1 = \frac{2\pi}{15}. \end{aligned}$$

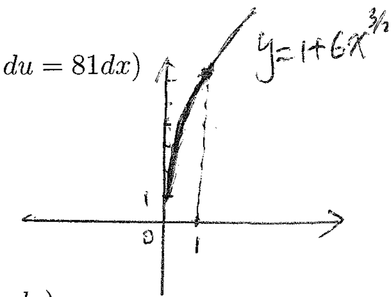
Method 2(Shell-Method).



$$\begin{aligned} V_3 &= \int_0^1 2\pi x \cdot [Top] \; dx \; - \int_0^1 2\pi x \cdot [Bottom] \; dx = 2\pi \int_0^1 x\sqrt{x} dx - 2\pi \int_0^1 x^2 \; dx \\ &= 2\pi \int_0^1 (x^{3/2} - x^2) dy = 2\pi \Big(\frac{2}{5}x^{5/2} - \frac{1}{3}x^3\Big) \Big|_0^1 = \frac{2\pi}{15}. \end{aligned}$$

Solution to Q4.

$$\begin{aligned} s &= \int_0^1 \sqrt{1+[f'(x)]^2}dx = \int_0^1 \sqrt{1+81x}dx \qquad \text{(substitute } u = 1+81x,\ du = 81dx) \\ &= \frac{1}{81}\int_1^{82} u^{1/2}du = \frac{164}{243}\sqrt{82} - \frac{2}{243}. \end{aligned}$$



Solution to Q5-a).

$$\begin{aligned} \int_0^\infty xe^{-x^2}dx &= \lim_{t\to\infty}\int_0^t xe^{-x^2}dx \qquad (\text{ substitute } u=-x^2,\ du=-2xdx) \\ &= -\frac{1}{2}\lim_{t\to\infty}\int_0^{-t^2} e^u du = -\frac{1}{2}\lim_{t\to\infty} e^u\Big|_0^{-t^2} \\ &= -\frac{1}{2}\lim_{t\to\infty}(e^{-t^2}-1) = \frac{1}{2}. \end{aligned}$$

So, it is convergent.

Solution to Q5-b).

$$\begin{aligned} \int_0^1 \frac{1}{x^2+x}dx &= \lim_{t\to 0^+}\int_t^1 \frac{1}{x^2+x}dx \\ &= \lim_{t\to 0^+}\int_t^1 \Big(\frac{1}{x}-\frac{1}{x+1}\Big)dx = \lim_{t\to 0^+}\Big(\ln|x|-\ln|x+1|\Big)\Big|_t^1 \\ &= \lim_{t\to 0^+}\Big((\ln 1-\ln 2)-(\ln t-\ln(t+1))\Big) = +\infty. \end{aligned}$$

So, it is divergent.