Champlain College – St.-Lambert

MATH 201-103-RE: Calculus I

Review Questions for Test # 2

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Questions

1. Find the derivative of the function.

   (a) $f(x) = -2x^{-3} + e^{-1}x^2 + \dfrac{1}{\pi x}$.

   (b) $f(x) = (1 + \sqrt{x})^2(x^2 - 1)^3$.

   (c) $f(x) = \dfrac{\sqrt{2x^2 + 1}}{(x^2 + 1)^2}$.

2. Find the second derivative of the function.

   (a) $f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$.

   (b) $f(x) = \sqrt[3]{2x^2 + 1}$.

3. y is a function of x given as follows implicitly. Find y'.

   (a) $x^2 - 4xy - y^2 = 12$.

   (b) $y^3 + 3x^2 = 3y$.

4. Using a differential to approximate $\sqrt[3]{26.8}$.

5. The total weekly cost in dollars incurred by Herald Media Corp. in producing x DVDs is given by the total cost function

$$C(x) = 2500 + 40x - 0.05x^2,$$

   and the price is determined as

$$p = 600 - 0.02x.$$

   (a) Find the marginal cost function, and the marginal cost when $x = 100$. Interpret your result.

(b) Find the average cost function and the marginal average cost function.

(c) Find the revenue function and the marginal revenue function.

(d) Fin the profit function and the marginal profit function.

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Solutions to Review Questions

1(a).

$$\begin{aligned} f'(x) &= \Big(-2x^{-3} + e^{-1}x^2 + \frac{1}{\pi x}\Big)' = \Big(-2x^{-3} + e^{-1}x^2 + \frac{1}{\pi}x^{-1}\Big)' \\ &= 6x^{-4} + 2e^{-1}x - \frac{1}{\pi}x^{-2}. \end{aligned}$$

1(b).

$$\begin{aligned} f'(x) &= \Big((1+\sqrt{x})^2\Big)'(x^2-1)^3 + (1+\sqrt{x})^2\Big((x^2-1)^3\Big)' \\ &\qquad \text{[by Product Rule]} \\ &= 2(1+\sqrt{x})^{2-1}(1+\sqrt{x})'(x^2-1)^3 + (1+\sqrt{x})^2 \cdot 3(x^2-1)^{3-1}(x^2-1)' \\ &\qquad \text{[by Chain Rule]} \\ &= \frac{1+\sqrt{x}}{\sqrt{x}}(x^2-1)^3 + 6x(1+\sqrt{x})^2(x^2-1)^2. \end{aligned}$$

1(c).

$$\begin{aligned}
f'(x) &= \frac{\left(\sqrt{2x^2+1}\right)'(x^2+1)^2 - \sqrt{2x^2+1}\left((x^2+1)^2\right)'}{[(x^2+1)^2]^2} \\
&\qquad\qquad\qquad\qquad\qquad\qquad \text{[by Quotient Rule]} \\
&= \frac{\frac{1}{2}(2x^2+1)^{\frac{1}{2}-1}(2x^2+1)'(x^2+1)^2 - \sqrt{2x^2+1}\cdot 2(x^2+1)^{2-1}(x^2+1)'}{(x^2+1)^4} \\
&\qquad\qquad\qquad\qquad\qquad\qquad \text{[by Chain Rule]} \\
&= \frac{2x(2x^2+1)^{-\frac{1}{2}}(x^2+1)^2 - 4x\sqrt{2x^2+1}(x^2+1)}{(x^2+1)^4} \\
&= \frac{2x(2x^2+1)^{-\frac{1}{2}}(x^2+1) - 4x\sqrt{2x^2+1}}{(x^2+1)^3} \\
&\qquad\qquad\qquad\qquad \text{[cancel out one factor } (x^2+1) \text{]} \\
&= \frac{\frac{2x(x^2+1)}{\sqrt{2x^2+1}} - 4x\sqrt{2x^2+1}}{(x^2+1)^3} \\
&= \frac{2x(x^2+1) - 4x(\sqrt{2x^2+1})^2}{\sqrt{2x^2+1}\ (x^2+1)^3} \\
&= -\frac{2x(3x^2+1)}{(x^2+1)^3\sqrt{2x^2+1}}.
\end{aligned}$$

2(a).

$$f'(x) = (x^{\frac{1}{2}} + x^{-\frac{1}{2}})' = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$$

and

$$f''(x) = \left(\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}\right)' = -\frac{1}{4}x^{-\frac{3}{2}} + \frac{3}{4}x^{-\frac{5}{2}}$$

2(b).

$$f'(x) = \left((2x^2+1)^{\frac{1}{3}}\right)' = \frac{1}{3}(2x^2+1)^{\frac{1}{3}-1}(2x^2+1)' = \frac{4}{3}x(2x^2+1)^{-\frac{2}{3}}$$

and by the product rule and the chain rule,

$$f''(x) = \left(\frac{4}{3}x(2x^2+1)^{-\frac{2}{3}}\right)' = \frac{4}{3}(2x^2+1)^{-\frac{2}{3}} - \frac{32}{9}x^2(2x^2+1)^{-\frac{5}{3}}.$$

3(a). By using the Product Rule and the Chain Rule to the equation, we have $2x - 4y - 4xy' - 2yy' = 0$, which gives $y' = \dfrac{x-2y}{2x+y}$.

3(b). By using the Product Rule and the Chain Rule to the equation, we have $3y^2y' + 6x = 3y'$, which gives $y' = \dfrac{2x}{1-y^2}$.

4. Let $f(x) = \sqrt[3]{x}$. Its derivative is $f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \dfrac{1}{3(\sqrt[3]{x})^2}$. Set $x_1 = 27$, $x_2 = 26.8$, $y_1 = f(x_1) = \sqrt[3]{27} = 3$ and $y_2 = f(x_2) = \sqrt[3]{26.8}$, as well as $\Delta x = x_2 - x_1 = 26.8 - 27 = -0.2$, and $\Delta y = y_2 - y_1 = \sqrt[3]{26.8} - 3$. Since

$$\Delta y \approx dy = f'(x_1)dx = f'(x_1)\Delta x,$$

and $f'(x_1) = f'(27) = \dfrac{1}{3(\sqrt[3]{27})^2} = \dfrac{1}{27}$, we have

$$\sqrt[3]{26.8} - 3 \approx f'(27) \cdot (-0.2) = -\frac{0.2}{27},$$

which implies

$$\sqrt[3]{26.8} \approx 3 - \frac{0.2}{27} = 2.99\overline{259}.$$

5(a). The marginal cost function is: $C'(x) = 40 - 0.1x$. The marginal cost at $x = 100$ is $C'(100) = 40 - 0.1 \times 100 = \$30$. This means that the actual cost for producing the 101st DVD is approximately \$30.

5(b). The average cost function is

$$\overline{C}(x) = \frac{C(x)}{x} = 40 - 0.05x + \frac{2500}{x},$$

and the marginal average cost function is

$$\overline{C}'(x) = -0.05 - \frac{2500}{x^2}.$$

5(c). The revenue function is

$$R(x) = p(x)x = (600 - 0.02x)x = 600x - 0.02x^2,$$

and the marginal revenue function is

$$R'(x) = 600 - 0.04x.$$

5(d). The profit function is

$$P(x) = R(x) - C(x) = (600x - 0.02x^2) - (2500 + 40x - 0.05x^2) = 0.03x^2 + 560x - 2500,$$

and the marginal revenue function is

$$P'(x) = R'(x) - C'(x) = 0.06x + 560.$$