Review 3

CHAMPLAIN COLLEGE – ST.-LAMBERT

Final Exam

Mathematics 201-103-ER
Calculus I
Instructors: *J. Chvatalova, M. Mei, M. Titcombe*

Date: Tuesday, December 19, 2006
Time: 9:00 a.m. - 12:00 a.m.

Name: ________________________________

ID #: ________________________________

REMARKS:

- Only small scientific calculators without graphing program are permitted.
- A sheet of formulas is included at the end of the exam.
- Read every question carefully and show your work for all questions.
- When evaluating limits, note that if a limit does not exist and equals to ∞ or $-\infty$ write this as your answer, otherwise just write *does not exist* or DNE.
- No cell phones are allowed during the exam.

Question	Mark	
1		24
2		20
3		7
4		7
5		10
6		10
7		10
8		12
TOTAL		100

1. Evaluate each of the following limits. MARKS (24)

(a) $\lim_{x \to 1} \dfrac{x-1}{2x^2-2}$

(b) $\lim_{x \to \infty} \dfrac{x(5x+1)}{1-2x^2}$

(c) $\lim_{x \to 0} \frac{e^x - 1}{4x}$

(d) $\lim_{x \to 2^-} \frac{|x-2|}{x-2}$

(e) $\lim_{x \to 0} \frac{\sin 2x}{4x}$

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(f) $\lim_{x \to 0^+} x^{x^3}$

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2. Find the derivative of each of the following functions. MARKS (20)

(a) $y = x^4 - 4\sqrt[4]{x} + \dfrac{1}{x^4}$

(b) $y = x^2 e^{x^2}$

(c) $y = \sin x^2 + (\sin x)^2$

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(d) $y = \dfrac{\tan 3x - e^{4x}}{x+1}$

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(e) $y = \dfrac{\ln x}{x^2}$

3. Find the constant c that makes g continuous on $(-\infty, \infty)$. <u>MARKS</u>(7)

$$g(x) = \begin{cases} x^2 - c^2, & \text{if } x < 4 \\ cx + 20, & \text{if } x \geq 4. \end{cases}$$

4. Use logarithmic differentiation to find the derivative of the function $y = x^{e^x}$. MARKS(7)

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5. Find the equation of the tangent line to the graph $x^2y - xy^2 + 6 = 0$ at the point $(1, 3)$. MARKS(10)

6. A ladder 5 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 3 ft from the wall? MARKS(10)

7. A function f has the following properties

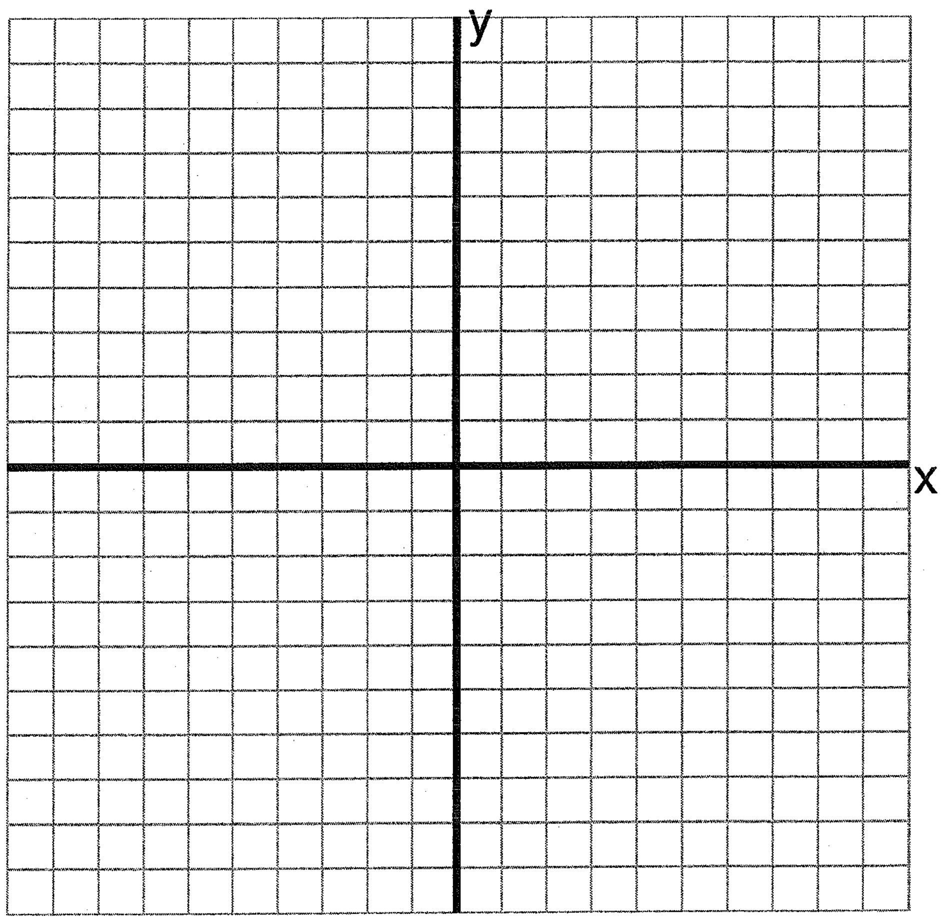
1). $\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to-\infty} f(x) = 0$, $\lim_{x\to-1^-} f(x) = -\infty$, $\lim_{x\to-1^+} f(x) = \infty$,

2). $f(\frac{1}{2}) = 0$, $f(2) = 2$, $f'(\frac{1}{2}) = f'(2) = 0$, $f''(1) = f''(3) = 0$,

3). $f'(x) > 0$ for $\frac{1}{2} < x < 2$, $f'(x) < 0$ for $x < \frac{1}{2}$ (but $x \neq -1$) and $x > 2$,

4). $f''(x) < 0$ for $x < -1$ and $1 < x < 3$, $f''(x) > 0$ for $-1 < x < 1$ and $x > 3$.

Sketch the graph. MARKS(10)



8. A company estimates that the cost (in dollars) of producing x items and the demand function are: $C(x) = 200 + 4000x - 150x^2 + x^3$ and $p(x) = 400 - 45x + \frac{50000}{x}$, respectively.

   (a) Find the revenue function and the profit function, respectively.

   (b) Find the marginal profit function.

   (c) At what production level will the profit be maximum? and what is the maximum profit?

MARKS(12)