MATH 203/2 FALL 2006
ASSIGNMENT 5 SOLUTIONS

A32

32. $\cos x = -\frac{1}{3}$, $\pi < x < \frac{3\pi}{2}$, i.e. in quadrant 3, so $\sin x$ is also negative and therefore:

$$\begin{aligned} \sin x &= -\sqrt{1-\cos^2 x} = -\frac{2\sqrt{2}}{3} \\ \tan x &= \frac{\sin x}{\cos x} = 2\sqrt{2} \\ \cot x &= \frac{1}{\tan x} = \frac{1}{2\sqrt{2}} \\ \sec x &= \frac{1}{\cos x} = -3 \\ \csc x &= \frac{1}{\sin x} = -\frac{3}{2\sqrt{2}} \end{aligned}$$

54. Prove

$$\sin^2 x - \sin^2 y = \sin(x+y)\sin(x-y)$$

Start with the right hand side

$$\begin{aligned} & \sin(x+y)\sin(x-y) \\ =\ & (\sin x\cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) \\ =\ & \sin^2 x \cos^2 y - \cos^2 x \sin^2 y \\ =\ & \sin^2 x\left(1-\sin^2 y\right) - \left(1-\sin^2 x\right)\sin^2 y \\ =\ & \sin^2 x - \sin^2 y \end{aligned}$$

which is the left hand side.

88. The area A of the triangle is

$$A = \frac{1}{2} \cdot \text{base} \cdot \text{height}$$

If we take the base to be the side a and the height to be h then

$$\frac{h}{b} = \sin\theta$$

whether $\theta < \pi/2$ (acute) or $\theta > \pi/2$ (obtuse), as is easy to see from a diagram. (If $\theta = \pi/2$ we don't have a triangle). Therefore

$$A = \frac{1}{2}ab$$

Section 1.6

68. **(a)** Let $\theta = \arctan 2$, so $\tan\theta = 2 \Rightarrow \sec^2\theta = 1+\tan^2\theta = 1+4 = 5 \Rightarrow \sec\theta = \sqrt{5} \Rightarrow \sec(\arctan 2) = \sec\theta = \sqrt{5}$.

(b) Let $\theta = \sin^{-1}\frac{5}{13}$. Then $\sin\theta = \frac{5}{13}$, so

$$\cos\left(2\sin^{-1}\frac{5}{13}\right) = \cos 2\theta = 1-2\sin^2\theta = 1-2\left(\frac{5}{13}\right)^2 = \frac{119}{169}.$$

72.

Let $y = \cos^{-1}x$. Then $\cos y = x \Rightarrow \sin y = \sqrt{1-x^2}$ since $0 \le y \le \pi$. So $\sin(2\cos^{-1}x) = \sin 2y = 2\sin y\cos y = 2x\sqrt{1-x^2}$.

Section 3.4

8. $y = e^u(\cos u + cu) \Rightarrow y' = e^u(-\sin u + c) + (\cos u + cu)e^u = e^u(\cos u - \sin u + cu + c)$

10. $y = \dfrac{1+\sin x}{x+\cos x} \Rightarrow$

$$y' = \frac{(x+\cos x)(\cos x)-(1+\sin x)(1-\sin x)}{(x+\cos x)^2} = \frac{x\cos x+\cos^2 x-(1-\sin^2 x)}{(x+\cos x)^2}$$

$$= \frac{x\cos x+\cos^2 x-(\cos^2 x)}{(x+\cos x)^2} = \frac{x\cos x}{(x+\cos x)^2}$$

16. Recall that if $y = fgh$, then $y' = f'gh + fg'h + fgh'$. $y = x\sin x\cos x \Rightarrow$
$$\frac{dy}{dx} = \sin x\cos x + x\cos x\cos x + x\sin x(-\sin x) = \sin x\cos x + x\cos^2 x - x\sin^2 x$$

18. $$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{(\cos x)(0)-1(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x} = \sec x\tan x$$

24. $y=\frac{1}{\sin x+\cos x} \Rightarrow y'=-\frac{\cos x-\sin x}{(\sin x+\cos x)^2}$ [Reciprocal Rule]. At $(0,1)$, $y'=-\frac{1-0}{(0+1)^2}=-1$, and an equation of the tangent line is $y-1=-1(x-0)$, or $y=-x+1$.

36.

$$\lim_{x\to 0}\frac{\sin 4x}{\sin 6x}=\lim_{x\to 0}\left(\frac{\sin 4x}{x}\cdot\frac{x}{\sin 6x}\right)=\lim_{x\to 0}\frac{4\sin 4x}{4x}\cdot\lim_{x\to 0}\frac{6x}{6\sin 6x}$$

$$=4\lim_{x\to 0}\frac{\sin 4x}{4x}\cdot\frac{1}{6}\lim_{x\to 0}\frac{6x}{\sin 6x}=4(1)\cdot\frac{1}{6}(1)=\frac{2}{3}$$