MATH 203/2 FALL 2006
ASSIGNMENT 2 SOLUTIONS

Section 2.2

8. **(a)** $\lim_{x\to 2} R(x)=-\infty$

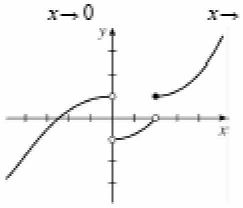
(b) $\lim_{x\to 5} R(x)=\infty$

(c) $\lim_{x\to -3^-} R(x)=-\infty$

(d) $\lim_{x\to -3^+} R(x)=\infty$

(e) The equations of the vertical asymptotes are $x=-3$, $x=2$, and $x=5$.

14. $\lim_{x\to 0^-} f(x)=1$, $\lim_{x\to 0^+} f(x)=-1$, $\lim_{x\to 2^-} f(x)=0$, $\lim_{x\to 2^+} f(x)=1$, $f(2)=1$, $f(0)$ is undefined



26. $\lim_{x\to 0} \frac{x-1}{x^2(x+2)} =-\infty$ since $x^2\to 0$ as $x\to 0$ and $\frac{x-1}{x^2(x+2)}<0$ for $0<x<1$ and for $-2<x<0$.

Section 2.3

14. $\lim_{x\to 4} \frac{x^2-4x}{x^2-3x-4} =\lim_{x\to 4} \frac{x(x-4)}{(x-4)(x+1)} =\lim_{x\to 4} \frac{x}{x+1} = \frac{4}{4+1} = \frac{4}{5}$

22.

$$\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} = \lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}\cdot\frac{\sqrt{1+h}+1}{\sqrt{1+h}+1} = \lim_{h\to 0} \frac{(1+h)-1}{h\left(\sqrt{1+h}+1\right)} = \lim_{h\to 0} \frac{h}{h\left(\sqrt{1+h}+1\right)}$$

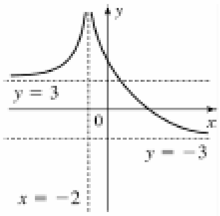
$$= \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

26. $\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t} \right) =\lim_{t\to 0} \frac{\left(t^2+t\right)-t}{t\left(t^2+t\right)} =\lim_{t\to 0} \frac{t^2}{t\cdot t(t+1)} =\lim_{t\to 0} \frac{1}{t+1} = \frac{1}{0+1} =1$

40. If $x<-4$, then $|x+4|=-(x+4)$, so $\lim_{x\to -4^-} \frac{|x+4|}{x+4} =\lim_{x\to -4^-} \frac{-(x+4)}{x+4} =\lim_{x\to -4^-} (-1)=-1.$

Section 2.6

8. $\lim_{x\to -2} f(x)=\infty$, $\lim_{x\to -\infty} f(x)=3$, $\lim_{x\to \infty} f(x)=-3$



12.

$$\lim_{x\to\infty}\sqrt{\frac{12x^3-5x+2}{1+4x^2+3x^3}}=\sqrt{\lim_{x\to\infty}\frac{12x^3-5x+2}{1+4x^2+3x^3}}$$ [Limit Law 11]

$$=\sqrt{\lim_{x\to\infty}\frac{12-5/x^2+2/x^3}{1/x^3+4/x+3}}$$ [divide by x^3]

$$=\sqrt{\frac{\lim_{x\to\infty}(12-5/x^2+2/x^3)}{\lim_{x\to\infty}(1/x^3+4/x+3)}}$$ [Limit Law 5]

$$=\sqrt{\frac{\lim_{x\to\infty}12-\lim_{x\to\infty}(5/x^2)+\lim_{x\to\infty}(2/x^3)}{\lim_{x\to\infty}(1/x^3)+\lim_{x\to\infty}(4/x)+\lim_{x\to\infty}3}}$$ [Limit Laws 1 and 2]

$$=\sqrt{\frac{12-5\lim_{x\to\infty}(1/x^2)+2\lim_{x\to\infty}(1/x^3)}{\lim_{x\to\infty}(1/x^3)+4\lim_{x\to\infty}(1/x)+3}}$$ [Limit Laws 7 and 3]

$$=\sqrt{\frac{12-5(0)+2(0)}{0+4(0)+3}}$$ [Theorem 5 of Section 2.5]

$$=\sqrt{\frac{12}{3}}=\sqrt{4}=2$$

16. $$\lim_{y\to\infty}\frac{2-3y^2}{5y^2+4y}=\lim_{y\to\infty}\frac{(2-3y^2)/y^2}{(5y^2+4y)/y^2}=\frac{\lim_{y\to\infty}(2/y^2-3)}{\lim_{y\to\infty}(5+4/y)}=\frac{2\lim_{y\to\infty}(1/y^2)-\lim_{y\to\infty}3}{\lim_{y\to\infty}5+4\lim_{y\to\infty}(1/y)}=\frac{2(0)-3}{5+4(0)}=-\frac{3}{5}$$

22.

$$\lim_{x\to-\infty}\frac{\sqrt{9x^6-x}}{x^3+1}=\lim_{x\to-\infty}\frac{\sqrt{9x^6-x}\,/x^3}{(x^3+1)/x^3}=\frac{\lim_{x\to-\infty}-\sqrt{(9x^6-x)/x^6}}{\lim_{x\to-\infty}(1+1/x^3)}$$ [since $x^3=-\sqrt{x^6}$ for $x<0$]

$$=\frac{\lim_{x\to-\infty}-\sqrt{9-1/x^5}}{\lim_{x\to-\infty}1+\lim_{x\to-\infty}(1/x^3)}=\frac{-\sqrt{\lim_{x\to-\infty}9-\lim_{x\to-\infty}(1/x^5)}}{1+0}$$

$$=-\sqrt{9-0}=-3$$