MATH 203/2 FALL 2006
ASSIGNMENT 1 SOLUTIONS

Section 1.1

2. **(a)** The point $(-4,-2)$ is on the graph of f, so $f(-4)=-2$. The point $(3,4)$ is on the graph of g, so $g(3)=4$.
(b) We are looking for the values of x for which the y –values are equal. The y –values for f and g are equal at the points $(-2,1)$ and $(2,2)$, so the desired values of x are -2 and 2.
(c) $f(x)=-1$ is equivalent to $y=-1$. When $y=-1$, we have $x=-3$ and $x=4$.
(d) As x increases from 0 to 4, y decreases from 3 to -1. Thus, f is decreasing on the interval $[0,4]$.
(e) The domain of f consists of all x –values on the graph of f. For this function, the domain is $-4\leq x\leq 4$, or $[-4,4]$. The range of f consists of all y –values on the graph of f. For this function, the range is $-2\leq y\leq 3$, or $[-2,3]$.
(f) The domain of g is $[-4,3]$ and the range is $[0.5,4]$.

6. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-2,2]$ and the range is $[-1,2]$.

20. A spherical balloon with radius $r+1$ has volume $V(r+1)=\frac{4}{3}\pi(r+1)^3=\frac{4}{3}\pi\left(r^3+3r^2+3r+1\right)$. We wish to find the amount of air needed to inflate the balloon from a radius of r to $r+1$. Hence, we need to find the difference $V(r+1)-V(r)=\frac{4}{3}\pi\left(r^3+3r^2+3r+1\right)-\frac{4}{3}\pi r^3=\frac{4}{3}\pi\left(3r^2+3r+1\right)$.

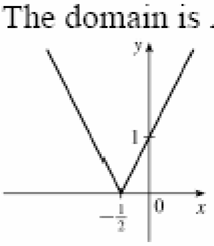
26. $g(u)=\sqrt{u}+\sqrt{4-u}$ is defined when $u\geq 0$ and $4-u\geq 0 \Leftrightarrow u\leq 4$. Thus, the domain is $0\leq u\leq 4=[0,4]$.

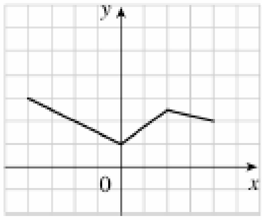
34.

$$F(x)=|2x+1| = \begin{cases} 2x+1 & \text{if } 2x+1\geq 0 \\ -(2x+1) & \text{if } 2x+1<1 \end{cases}$$

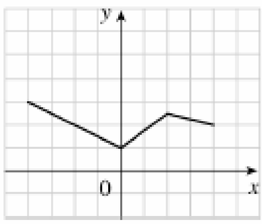
$$= \begin{cases} 2x+1 & \text{if } x\geq -\frac{1}{2} \\ -2x-1 & \text{if } x<-\frac{1}{2} \end{cases}$$

The domain is R, or $(-\infty,\infty)$.





The point $(2,1)$ on the graph of f corresponds to the point $\left(2,-\frac{1}{2}\cdot 1+3\right)=(2,2.5)$.



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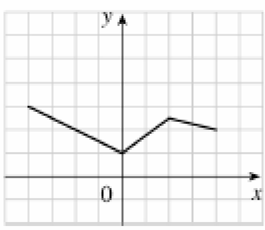
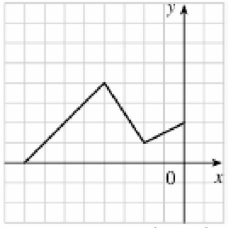


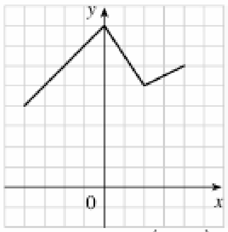
The point $(2,1)$ on the graph of f corresponds to the point $\left(2,-\frac{1}{2}\cdot 1+3\right)=(2,2.5)$.

4. **(a)** To graph $y = f(x+4)$ we shift the graph of f, 4 units to the left.



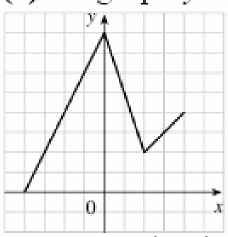
The point $(2,1)$ on the graph of f corresponds to the point $(2-4,1) = (-2,1)$.

(b) To graph $y = f(x)+4$ we shift the graph of f, 4 units upward.



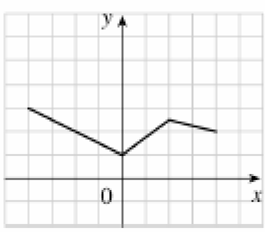
The point $(2,1)$ on the graph of f corresponds to the point $(2,1+4) = (2,5)$.

(c) To graph $y = 2f(x)$ we stretch the graph of f vertically by a factor of 2.



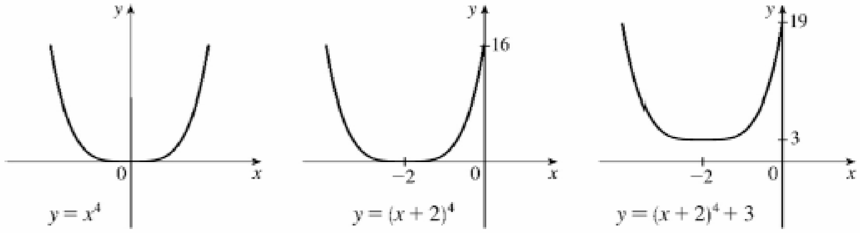
The point $(2,1)$ on the graph of f corresponds to the point $(2,2\cdot 1) = (2,2)$.

(d) To graph $y = -\frac{1}{2} f(x)+3$, we shrink the graph of f vertically by a factor of 2, then reflect the resulting graph about the x-axis, then shift the resulting graph 3 units upward.



The point $(2,1)$ on the graph of f corresponds to the point $\left(2, -\frac{1}{2}\cdot 1+3\right) = (2,2.5)$.

18. $y=(x+2)^4+3$: Start with the graph of $y=x^4$, shift 2 units to the left, and then shift 3 units upward.



40. $f(x)=\sqrt{2x+3}$, $D=\left\{ x \mid x \geq -\frac{3}{2} \right\}$; $g(x)=x^2+1$, $D=R$.

$(f \circ g)(x)=f(x^2+1)=\sqrt{2(x^2+1)+3}=\sqrt{2x^2+5}$, $D=R$.

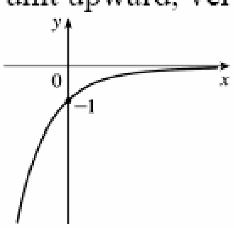
$(g \circ f)(x)=g\left(\sqrt{2x+3}\right)=\left(\sqrt{2x+3}\right)^2+1=(2x+3)+1=2x+4$, $D=\left\{ x \mid x \geq -\frac{3}{2} \right\}$

$(f \circ f)(x)=f\left(\sqrt{2x+3}\right)=\sqrt{2\left(\sqrt{2x+3}\right)+3}=\sqrt{2\sqrt{2x+3}+3}$, $D=\left\{ x \mid x \geq -\frac{3}{2} \right\}$.

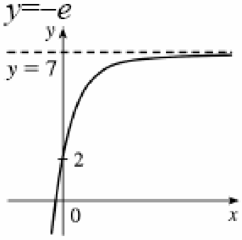
$(g \circ g)(x)=g(x^2+1)=(x^2+1)^2+1=(x^4+2x^2+1)+1=x^4+2x^2+2$, $D=R$.

Section 1.5

12. We start with the graph of $y=e^x$ (Figure 13), reflect it about the y −axis, and then about the x −axis (or just rotate 180° to handle both reflections) to obtain the graph of $y=-e^{-x}$. Now shift this graph 1 unit upward, vertically stretch by a factor of 5 , and then shift 2 units upward.



$y=-e^{-x}$



$y=2+5(1-e^{-x})$

14. **(a)** This reflection consists of first reflecting the graph about the x −axis (giving the graph with equation $y=-e^x$) and then shifting this graph $2 \cdot 4=8$ units upward. So the equation is $y=-e^x+8$.
(b) This reflection consists of first reflecting the graph about the y −axis (giving the graph with equation $y=e^{-x}$) and then shifting this graph $2 \cdot 2=4$ units to the right. So the equation is $y=e^{-(x-4)}$.

Section 1.6

20. **(a)** f is $1-1$ because it passes the Horizontal Line Test.

(b) Domain of $f=[-3,3]=$ Range of f^{-1}. Range of $f=[-2,2]=$ Domain of f^{-1}.

(c) Since $f(-2)=1$, $f^{-1}(1)=-2$.

28. $y=\dfrac{1+e^x}{1-e^x} \Rightarrow y-ye^x=1+e^x \Rightarrow y-1=ye^x+e^x \Rightarrow y-1=e^x(y+1) \Rightarrow$

$e^x=\dfrac{y-1}{y+1} \Rightarrow x=\ln\left(\dfrac{y-1}{y+1}\right)$. Interchange x and y : $y=\ln\left(\dfrac{x-1}{x+1}\right)$. So $f^{-1}(x)=\ln\left(\dfrac{x-1}{x+1}\right)$.

Note that the domain of f^{-1} is $|x|>1$.

36. **(a)** $\log_8 2=\dfrac{1}{3}$ since $8^{1/3}=2$.

(b) $\ln e^{\sqrt{2}}=\sqrt{2}$

38. **(a)** $2^{\left(\log_2 3+\log_2 5\right)}=2^{\log_2 15}=15$ [*Or:* $2^{\left(\log_2 3+\log_2 5\right)}=2^{\log_2 3}\cdot 2^{\log_2 5}=3\cdot 5=15$]

(b) $e^{3\ln 2}=e^{\ln(2^3)}=e^{\ln 8}=8$ [*Or:* $e^{3\ln 2}=\left(e^{\ln 2}\right)^3=2^3=8$]

40. $\ln x+a\ln y-b\ln z=\ln x+\ln y^a-\ln z^b=\ln(x\cdot y^a)-\ln z^b=\ln(xy^a/z^b)$

50. **(a)** $e^{2x+3}-7=0 \Rightarrow e^{2x+3}=7 \Rightarrow 2x+3=\ln 7 \Rightarrow 2x=\ln 7-3 \Rightarrow x=\dfrac{1}{2}(\ln 7-3)$

(b) $\ln(5-2x)=-3 \Rightarrow 5-2x=e^{-3} \Rightarrow 2x=5-e^{-3} \Rightarrow x=\dfrac{1}{2}(5-e^{-3})$