

CHAMPLAIN COLLEGE – ST.-LAMBERT

Final Exam

Mathematics 201-103-ER

Calculus I (Commerce Study)

Instructor: *Dr. Ming Mei*

Name: _____

ID #: _____

REMARKS:

- Only small scientific calculators without graphing program are permitted.
- Read every question carefully and show your work for all questions.
- When evaluating limits, note that if a limit does not exist and equals to ∞ or $-\infty$ write this as your answer, otherwise just write *does not exist* or DNE.
- No cell phones are allowed during the exam.

Question	Mark	
1		24
2		24
3		10
4		10
5		20
6		12
TOTAL		100

1. Evaluate each of the following limits.

MARKS (24)

(a) $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}$

(b) $\lim_{x \rightarrow \infty} \frac{3x^2 + x}{1 - 4x^3}$

(c) $\lim_{x \rightarrow 1^-} \frac{|x - 1|}{x - 1}$

2. Find the derivative of each of the following functions.

MARKS (24)

(a) $y = (x^2 + 1)^2(2x^2 + 5x)$

(b) $y = x e^{x^2}$

(c) $y = \frac{\ln(2x^2 + 1)}{x + 1}$

3. Find the constant c that makes g continuous on $(-\infty, \infty)$.

MARKS(10)

$$g(x) = \begin{cases} 2x^2 + c, & \text{if } x < 2 \\ \frac{c^2}{2}x + 6, & \text{if } x \geq 2. \end{cases}$$

4. Find the equation of the tangent line to the curve of $x^2y^3 + xy^2 - 2x^2 = 0$ at the point $(1, 1)$.

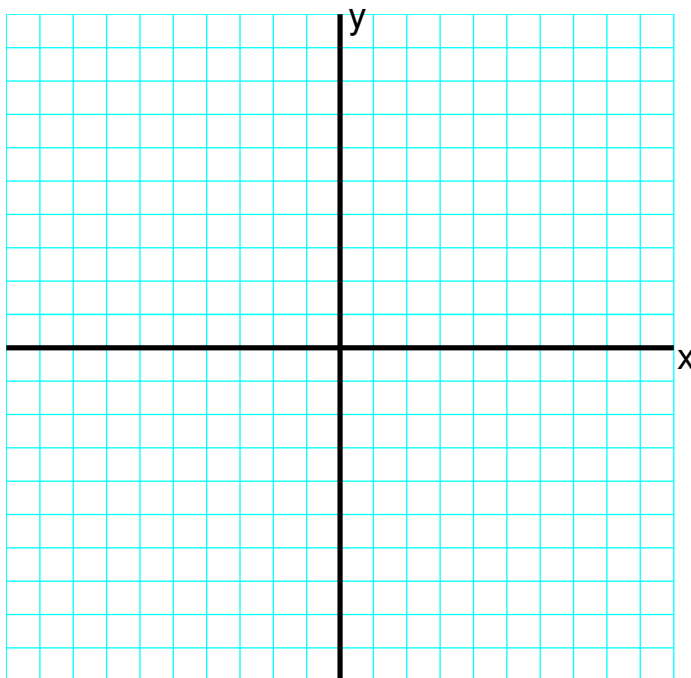
MARKS(10)

5. A function f has the following properties

- ♠ $f(-6) = 2, \quad f(-4) = 4, \quad f(0) = 0, \quad f(4) = -4, \quad f(6) = -2,$
- ♠ $f'(-4) = f'(4) = 0, \quad f''(-6) = f''(0) = f''(6) = 0,$
- ♠ $\lim_{x \rightarrow -\infty} f(x) = 0, \quad \lim_{x \rightarrow +\infty} f(x) = 0,$
- ♠ $f'(x) > 0$ in $(-\infty, -4) \cup (4, \infty), \quad f'(x) < 0$ in $(-4, 4),$
- ♠ $f''(x) > 0$ in $(-\infty, -6) \cup (0, 6), \quad f''(x) < 0$ in $(-6, 0) \cup (6, \infty).$

- (a) Find the intervals where $f(x)$ is increasing and where it is decreasing;
- (b) Find the relative extrema, if any;
- (c) Find the intervals where $f(x)$ is concave upward and where it is concave downward;
- (d) Point out the inflection points, if any;
- (e) Sketch the graph of $f(x)$.

MARKS(20)



6. The total weekly cost in dollars incurred by SONY Corp. in producing x HD TVs is given by the total cost function $C(x) = 4000 + 104x^2 - x^3$ and the price is determined as $p(x) = 3600 - x$.
- (a) Find the revenue function and the profit function, respectively.
 - (b) Find the marginal profit function.
 - (c) At what production level will the profit be maximum? and what is the maximum profit?

MARKS(12)