Nth term test, comparison

1. Suppose you know that the series $\sum_{k=1}^{\infty} a_k$ converges. Let $s_n = a_1 + a_2 + \cdots + a_n$, the n-th partial sum. For each of the following, determine whether the statement must be true, must be false, or could be either true or false.

   (a) $\lim_{n\to\infty} a_n = 0$.

   (b) $\lim_{n\to\infty} s_n = 0$.

   (c) $\sum_{k=5}^{\infty} a_k$ converges.

2. Suppose $a_1, a_2, \ldots$ are numbers and $\lim_{k\to\infty} a_k = 0$. Does it necessarily follow that the infinite series $\sum_{k=1}^{\infty} a_k$ converges?

3. The series $\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ is known as the *harmonic series.*

(a) What does the Nth Term Test tell you about the harmonic series?

(b) Does the harmonic series converge or diverge?

4. (a) Does the series $1+\frac{1}{2}+\frac{1}{2}+\underbrace{\frac{1}{4}+\cdots+\frac{1}{4}}_{\text{4 terms}}+\underbrace{\frac{1}{8}+\cdots+\frac{1}{8}}_{\text{8 terms}}+\underbrace{\frac{1}{16}+\cdots+\frac{1}{16}}_{\text{16 terms}}+\cdots$ converge or diverge?

(b) Does the series $\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\underbrace{\frac{1}{8}+\cdots+\frac{1}{8}}_{\text{4 terms}}+\underbrace{\frac{1}{16}+\cdots+\frac{1}{16}}_{\text{8 terms}}+\underbrace{\frac{1}{32}+\cdots+\frac{1}{32}}_{\text{16 terms}}+\cdots$ converge or diverge?

5. Does the series $\sum_{k=1}^{\infty} \frac{1}{2^k + k}$ converge or diverge?

6. Does the series $\sum_{k=10^{10}}^{\infty} \frac{1}{100000000k}$ converge or diverge?

7. Decide whether each of the following series converges, and use the Comparison Test to justify your answer mathematically.

(a) $\sum_{k=100}^{\infty} \frac{5 + 3\sin k}{k}$.

(b) $\sum_{k=37}^{\infty} \frac{5 + 3\sin k}{2^k}$.