CHAMPLAIN COLLEGE – ST.-LAMBERT

Review Questions for Test 1

1. Which of the following numbers are natural, integer, rational, irrational?

$$-43, \quad -43.43, \quad \frac{43}{2}, \quad \frac{86}{43}, \quad \sqrt{9}, \quad \pi, \quad e, \quad \frac{2\pi}{\pi}, \quad 2^{-0.5}.$$

2. Simplify: $\dfrac{-12x^3y^{-4}}{8x^7y^{-6}}$.

3. Solve:
   (a) $-\dfrac{5}{3}x + \dfrac{7}{3} = -5 + 2x$.
   (b) $|2x + 5| = |x - 9|$.

4. Solve:
   (a) $|2x - 1| < 12$
   (b) $|2 - 3(x - 2)| > 6$.

5. A piece of rope $27m$ long is cut into two pieces so that one piece is four-fifth as long as the other. Find the length of each piece.

6. Find the intersection and the union of $\{1, 2, 5, 6, 9\}$ and $\{1, 3, 5, 9\}$.

7. Let $f(x) = \dfrac{2}{(x-2)(x+1)}$. Find its domain.

8. Let a line pass through the points $(1, 1)$ and $(3, 5)$. Find the slope and the equation of the line, respectively.

9. Let $x + 2y = 1$ be the equation of the line l_1, the line l_2 be parallel to the line l_1 and pass through the point $P(1, 1)$, and the line l_3 be perpendicular to the line l_1 and pass through the same point $P(1, 1)$. Find the equation for the lines l_2 and l_3, respectively.

Solutions to Review Questions for Test 1

Solution to Q.1: Since $\dfrac{86}{43} = 2$, $\sqrt{9} = 3$, $\dfrac{2\pi}{\pi} = 2$ and $2^{-0.5} = \dfrac{1}{2^0.5} = \dfrac{1}{\sqrt{2}}$, we then know:

the natural numbers: $\frac{86}{43}$, $\sqrt{9}$, $\frac{2\pi}{\pi}$,

the integer numbers: -43, $\frac{86}{43}$, $\sqrt{9}$, $\frac{2\pi}{\pi}$,

the rational numbers: -43, -43.43, $\frac{43}{2}$, $\frac{86}{43}$, $\sqrt{9}$, $\frac{2\pi}{\pi}$,

and the irrational numbers: π, e, $2^{-0.5}$.

Solution to Q.2:

$$\frac{-12x^3y^{-4}}{8x^7y^{-6}} = -\frac{12}{8}\frac{x^3}{x^7}\frac{y^{-4}}{y^{-6}} = -\frac{3}{2}x^{3-7}y^{(-4)-(-6)} = -\frac{3}{2}x^{-4}y^2 = -\frac{3y^2}{x^4}.$$

Solution to Q.3(a): Adding 5 and $\frac{5}{3}x$ to both sides of the equation, we have

$$\frac{7}{3} + 5 = 2x + \frac{5}{3}x,$$

namely,

$$\frac{7}{3} + \frac{15}{3} = \left(\frac{6}{3} + \frac{5}{3}\right)x,$$

i.e.,

$$\frac{11}{3}x = \frac{22}{3}.$$

Multiplying both sides of the equation by 3 and dividing by 11, respectively, we further have

$$x = \frac{22}{3} \cdot \frac{3}{11} = 2.$$

Solution to Q.3(b): $|2x+5| = |x-9|$ is equivalent to $2x+5 = \pm(x-9)$.

For $2x+5 = x-9$, by subtracting x and 5 from both sides of the equation, we obtain

$$2x - x = -9 - 5, \ \textit{i.e.},\ x = -14.$$

For $2x+5 = -(x-9)$, namely, $2x+5 = -x+9$, by adding x to the equation and subtracting 5 from both sides of the equation, respectively, we obtain

$$2x + x = 9 - 5,\ \textit{i.e.},\ 3x = 4, \quad \text{that is,}\ \ x = \frac{4}{3}.$$

So, the solutions for the equation are $x = -14$ and $x = \frac{4}{3}$.

Solution to Q.4(a): $|2x-1| < 12$ is equivalent to

$$-12 < 2x - 1 < 12, \quad -12 + 1 < 2x < 12 + 1, \quad -\frac{11}{2} < x < \frac{13}{2}.$$

So, the solution is: $-\frac{11}{2} < x < \frac{13}{2}$, namely, x is in $(-\frac{11}{2}, \frac{13}{2})$.

Solution to Q.4(b): $|2-3(x-2)| > 6$ can be simplified as $|2-3x+6| > 6$, i.e., $|8-3x| > 6$, which is equivalent to $8-3x > 6$ or $8-3x < -6$.

For $8-3x > 6$, we have $8-3x+3x > 6+3x$, $8 > 6+3x$, $8-6 > 6+3x-6$, $2 > 3x$, $\frac{2}{3} > x$.

For For $8-3x < -6$, we have $8-3x+3x < -6+3x$, $8 < -6+3x$, $8+6 < -6+3x+6$, $2 > 3x$, $\frac{14}{3} < x$.

So, the solutions are, either $x < \frac{2}{3}$ or $x > \frac{14}{3}$, namely, $x \in (-\infty, \frac{2}{3}) \cup (\frac{14}{3}, \infty)$.

Solution to Q.5: Let the first piece of rope be x meter long. Since the second piece of rope is four-fifth as long as the first piece, we then know that the second piece of rope is $\frac{4}{5}x$. Notice that, the total length of two pieces is $27m$, namely,

$$\text{the length of the first piece} + \text{the length of the second piece} = 27m,$$

we have the following equation

$$x + \frac{4}{5}x = 27,$$

which can be solved as

$$\frac{5}{5}x + \frac{4}{5}x = 27, \qquad \frac{5+4}{5}x = 27, \qquad \frac{9}{5}x = 27, \qquad x = 27 \cdot \frac{5}{9} = 15.$$

So, the first piece of rope is $15m$ and the second piece of rope is $\frac{4}{5} \cdot 15m = 12m$.

Solution to Q.6: The intersection is $\{1, 5, 9\}$ and the union is $\{1, 2, 3, 5, 6, 9\}$.

Solution to Q.7: To let the function make sense, we need to restrict x such that the denominator is non-zero,

$$(x-2)(x+1) \neq 0,$$

namely,

$$x - 2 \neq 0 \quad \text{and} \quad x + 1 \neq 0,$$

which gives

$$x \neq 2 \quad \text{and} \quad x \neq -1.$$

So, the domain is

$$D = R - \{-1, 2\} = (-\infty, -1) \cup (-1, 2) \cup (2, \infty).$$

Solution to Q.8: Let the equation of the line be $y = mx + b$. The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{3-1} = 2.$$

Since the point $P_1(1, 1)$ is on the line, we plot $x = 1$ and $y = 1$ to have

$$1 = m \cdot 1 + b = 2 \cdot 1 + b, \quad b = 1 - 2 = -1.$$

So, the equation of the line is $y = 2x - 1$.

Solution to Q.9: For line l_1: $x + 2y = 1$, it can be reduced to $y = -\frac{1}{2}x + 1$. So the slope of the line l_1 is

$$m_1 = -\frac{1}{2}.$$

Since the line l_2 is parallel to l_1, then the slope of l_2 is same to m_1, i.e.,

$$m_2 = m_1 = -\frac{1}{2}.$$

Let the equation of the line l_2 be $y = m_2 x + b_2$. Notice that, the point $P(1,1)$ is on the line, then we have

$$1 = m_2 \cdot 1 + b_2 = -\frac{1}{2} + b_2, \quad \text{which gives} \;\; b_2 = \frac{3}{2}.$$

So the equation of the line l_2 is

$$y = -\frac{1}{2}x + \frac{3}{2}.$$

For the line l_3, since it is perpendicular to l_1, then its slope m_3 satisfies

$$m_1 \cdot m_3 = -1,$$

which gives

$$m_3 = -\frac{1}{m_1} = 1\frac{1}{-\frac{1}{2}} = 2.$$

Let the equation of the line l_3 be $y = m_3 x + b_3$, as showed before, the point $P(1,1)$ on the line l_3 implies

$$1 = m_3 \cdot 1 + b_3 = 2 + b_3, \quad \textit{i.e.}, \;\; b_3 = -1.$$

So the equation of the line l_3 is $y = 2x - 1$.