$$\mathrm{T}(\beta)=\mathrm{T}(\langle\alpha\rangle(p)) \ . \tag{5}$$

The intuitive idea behind the operation $\square$, called *function-replacement*, is that $\alpha\square_p\beta$ is the function obtained by evaluating, at the place p and only at that place, the function-variable $\langle\alpha\rangle(p)$ as the composite function β. The condition (5) says that β is of the same type as $\langle\alpha\rangle(p)$, and therefore, the said evaluation is possible.

Given $\alpha\in A$ and $p\in|\langle\alpha\rangle|$, let $f=\langle\alpha\rangle(p)\in L$. Then α can be written in the form

$$\alpha=\alpha'\circ_q f(\alpha_1, \ldots, \alpha_n) \ , \tag{6}$$

where $\alpha', \alpha_1, \ldots, \alpha_n\in A$, and q is a suitable place $q\in\mathrm{s}(\alpha')$. Note that if f occurs in more than one place in α, then this *decomposition at* f of α is not unique; however, we have in mind the *decomposition of* α *at the place* p, in which f "stands for the occurrence at p". What these obscure words mean is intuitively clear, and will be made precise in section 5. The notation $f(\alpha_1, \ldots, \alpha_n)$ follows the term-representation explained above; it is, structurally, a repeated (or *simultaneous*, because of the presence of an appropriate commutative law) composition, as it was also indicated above.

Now, suppose, that, in addition, $\beta\in A$ such that (5). Let
We put

$$\alpha\square_p\beta \ \overline{\mathrm{def}} \ \alpha'\circ_q\beta(\alpha_1, \ldots, \alpha_n) \ . \tag{7}$$

Here, $\beta(\alpha_1, \ldots, \alpha_n)=\beta(\alpha_1/1, \ldots, \alpha_n/n)$ is simultaneous composition. $\mathrm{T}(\beta)=\mathrm{T}(f)$ implies that $\mathrm{s}(\beta)=\mathrm{s}(f)$, and so $\mathrm{t}(\alpha_i)=\mathrm{s}(f)(i)=\mathrm{s}(\beta)(i)$, which makes the term $\beta(\alpha_1, \ldots, \alpha_n)$ well-defined; but also, $\mathrm{T}(\beta)=\mathrm{T}(f)$ implies that $\mathrm{t}(\beta)=\mathrm{t}(f)$, which ensures that $\mathrm{t}(\beta(\alpha_1, \ldots, \alpha_n))=\mathrm{t}(\beta)=\mathrm{t}(f)=\mathrm{s}(\alpha')(q)$, and thus, the composition at q is well-defined.

Let us see how this works for the examples of 3-pd's ($k=2$) in the previous subsection. We are going to make the discussion easier to follow, by replacing the place-number p by the symbol which occurs at p in the given term; since the terms in the examples are *separated*, that is, have no repeated occurrences of symbols, this will not introduce ambiguity. Note that under this convention, with $f=(\mathrm{s}\alpha)(p)=\mathrm{t}\alpha$, (4) becomes

$$d(\alpha \circ_f \beta) = (d\alpha) \square_f (d\beta) \ ;$$

and the role of θ disappears (of course, for the general, non-separated case, the said simplification is not valid).

The 3-pd φ introduced in the previuos subsection can be written in the following two ways:

$$\varphi = (y \circ_i (w \circ_h v)) \circ_b x = (y \circ_b x) \circ_i (w \circ_h v)$$

(compare (3')). Let us go through the definition of the domain of each of the constituent 3-pd's here.

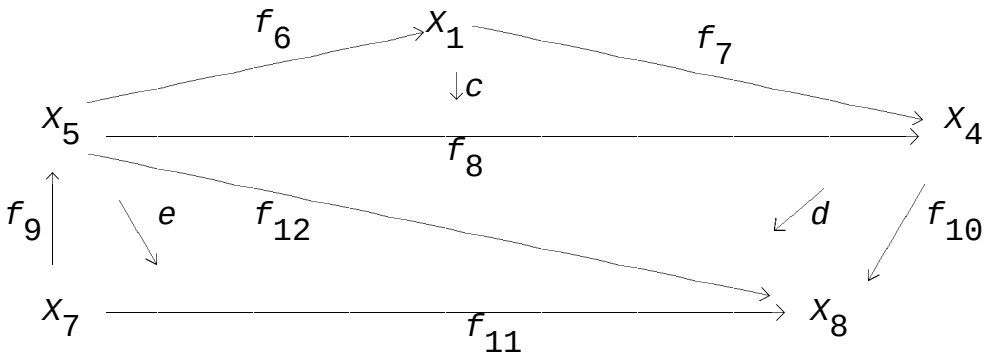
$$d(w \circ_h v) = (dw) \square_h (dv) = \eta \square_h \beta \ .$$

The decomposition of η at h has $\eta' = 1_{f_{11}}$ (we are writing η' for what was α' in the general case (6)); that is, now α' can be ignored in (6) and (7). (7) gives

$$\xi \ \bar{\bar{\text{def}}} \ \eta \square_h \beta = \beta(f_{10}/f_{10}, c(f_7, f_6)/f_8, f_9) = e(d(f_{10}, c(f_7, f_6)), f_9) \ ;$$

that is,

ξ :



ξ is obtained by replacing h with β as it should.

Next,

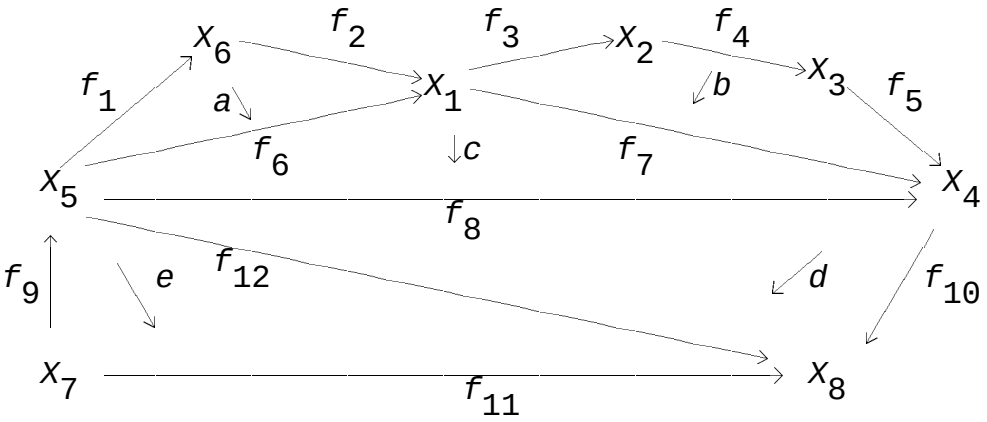
$$\mathrm{d}(y \circ_i (w \circ_h v)) = (\mathrm{d}y) \square \circ_i \mathrm{d}(w \circ_h v)) = \lambda \square_i \xi \ .$$

i is again the head-operation in λ, and so

$$\zeta \ \bar{\mathrm{def}} \ \lambda \square_i \xi = \xi(f_{10}/f_{10},\ b(f_5, f_6, f_3)/f_7,\ a(f_2, f_1)/f_6,\ f_9/f_9)$$
$$e(d(f_{10},\ c(b(f_5, f_6, f_3),\ a(f_2, f_1)),\ f_9) \ ,$$

that is,

ζ :



Note that ζ is the result of replacing i by ξ in λ.

Finally,

$$\mathrm{d}\varphi = \mathrm{d}(y \circ_i (w \circ_h v)) \square_b \mathrm{d}x = \zeta \square_b \delta = \zeta \square_b b = \zeta \ ;$$

note that $\delta = b$, and when b is replaced by b, nothing happens. Of course, $\zeta = \gamma$, for our initial γ, so this calculation confirms what we said "geometrically" about φ and γ.

Let us look at the other way of expressing φ. We have

$$\mathrm{d}(y \circ_b x) = \mathrm{d}y = \lambda \ ,$$

for the same reason as in the preceding case. $\mathrm{d}(w\circ_h v)=\xi$ was calculated above. Then

$$\mathrm{d}\varphi=\mathrm{d}(y\circ_b x)\square_i\mathrm{d}(w\circ_h v)=\lambda\square_i\xi=\zeta=\gamma,$$

as it should be the case.

In this subsection, we described the way the domain-function $\mathrm{d}:P_{k+1}\to P_k$ is actually calculated, and saw that, in some examples at least, it agrees with the geometric intuition. However, thereby the problem of defininig d is far from resolved. For instance, it is not clear that, in general, (4) is a compatible way of determining $\mathrm{d}\gamma$ for $\gamma\in P_{k+1}$, ; usually, γ can be written in more than one way as $\gamma=\alpha\circ_p\beta$, and we must see that the corresponding right-hand side expressions for $\mathrm{d}\gamma$ give the same result. There are other problems too. E.g., we have to see that if in (4), the left side is well-defined, so is the right side. Also note that we have not made any reference yet to the fact that d and c on C_{k+1} are determined so that the globularity condition (2) is satisfied. It is worth noting that that condition refers, besides d on P_{k+1}, also to d as defined on P_k. This suggests that d_k *must be defined recursively in* k.

1.5. Generalizing multicategories

The operation $\square$ used in the last subsection looks like a multicategory operation. Let us start with $\boldsymbol{C}$, a free multicategory on $\mathcal{L}$ as we had in the last subsection for the purposes of defining the operation $\square$ on the arrows of $\boldsymbol{C}$; let's use the same accompanying notation. We are going to define $\boldsymbol{D}$, a new multicategory, albeit in a somewhat generalized sense with respect to what we had above. $\boldsymbol{D}$ is called the *multicategory of function-replacement*. The arrows of $\boldsymbol{D}$ are the same as those of $\boldsymbol{C}$: $\mathrm{A}(\boldsymbol{D})=\mathrm{A}(\boldsymbol{C})=A$. The idea is to consider each $\alpha\in A$ to be a function not of its variable-occurrences, but of its function-symbol occurrences. The objects of $\boldsymbol{D}$ are pairs $(\vec{X};Y)$ where $\vec{X}\in O^*$ is a tuple of objects of $\boldsymbol{C}$, and Y is a single object: $\mathrm{O}(\boldsymbol{D})=O^*\times O$. If $\langle\alpha\rangle=\langle f_1,\ldots,f_n\rangle$, then, by definition, $\mathrm{s}_{\boldsymbol{D}}(\alpha)=\mathrm{S}(\alpha)\stackrel{\text{def}}{=}\langle \mathrm{T}f_1,\ldots,\mathrm{T}f_n\rangle$ and $\mathrm{t}_{\boldsymbol{D}}(\alpha)\stackrel{\text{def}}{=}\mathrm{T}(\alpha)=(\mathrm{s}\alpha,\mathrm{t}\alpha)$. The operation $\circ_{\boldsymbol{D}}$ is defined to be the operation $\square$ explained in the previous subsection.

Before we say more on to what extent $\mathbf{D}$ is a multicategory, let us point out in what aspect it fails to be one.

Consider a language $\mathcal{L}$ in which we have sorts U, V, W, X, Y and function-symbols

$$f:\langle U, V\rangle \to W\,,\ g:\langle X\rangle \to U\,,\ h:\langle U, Y\rangle \to W\,,\ i:\langle V\rangle \to Y\,. \tag{8}$$

Let $\beta=f(g(X), V)$, $\alpha=h(U, i(V))$, terms in $\mathrm{A}(\mathcal{L})$. We have $\alpha:\langle U, V\rangle \to W$, thus $\mathrm{T}(\alpha)=\mathrm{T}(f)$, and so $\beta\Box_1\alpha=\beta\Box_f\alpha$ is well-defined. Now, we have $\beta=1_W\circ_1 f(g(X), V)$ as the decomposition of β at 1 (at f), so

$$\beta\Box_1\alpha = 1_W\circ_1\alpha(g(X)/U, V/V) = \alpha(g(X)/U, V/V) = h(g(X), i(V))\,.$$

Also,

$$\langle\beta\rangle=\langle f, g\rangle\,,\ \mathrm{S}(\beta)=\langle \mathrm{T}f, \mathrm{T}g\rangle=\langle(\langle U, V\rangle; W), (\langle X\rangle; U)\rangle\,,$$
$$\langle\alpha\rangle=\langle h, i\rangle\,,\ \mathrm{S}(\alpha)=\langle \mathrm{T}h, \mathrm{T}i\rangle=\langle(\langle U, Y\rangle; W), (\langle V\rangle; Y)\rangle\,,$$
$$\langle\beta\Box_1\alpha\rangle=\langle h, g, i\rangle\,,$$
$$\mathrm{S}(\beta\Box_1\alpha)=\langle \mathrm{T}h, \mathrm{T}g, \mathrm{T}i\rangle=\langle(\langle U, Y\rangle; W), (\langle X\rangle; U), (\langle V\rangle; Y)\rangle\,.$$

In a Lambek multicategory $\mathbf{E}$, if $\mathrm{s}_{\mathbf{E}}(\beta)=\langle b_1, \ldots, b_n\rangle$, $\mathrm{s}_{\mathbf{E}}(\alpha)=\langle a_1, \ldots, a_m\rangle$, then for $\beta\circ_p\alpha=\beta\circ_p^{(\mathbf{E})}\alpha$, we have

$$\mathrm{s}_{\mathbf{E}}(\beta\circ_p\alpha)=\langle b_1, \ldots, b_{p-1}, a_1, \ldots, a_m, b_{p+1}, \ldots, b_n\rangle\,;$$

$\mathrm{s}_{\mathbf{E}}(\alpha)$ is inserted in the place of a_p; this is what we mean by *standard amalgamation* of the sources. The operation $\Box$ is much like a multicategory composition, except for the standard amalgamation. If $\mathbf{D}$ had standard amalgamation, $\mathrm{S}(\beta\Box_1\alpha)$ would have to be the result of inserting $\langle \mathrm{T}h, \mathrm{T}i\rangle$ into $\langle \mathrm{T}f, \mathrm{T}g\rangle$ in the place of $\mathrm{T}f$, resulting in $\langle \mathrm{T}h, \mathrm{T}i, \mathrm{T}g\rangle$; but $\mathrm{S}(\beta\Box_1\alpha)$ is, rather, $\langle \mathrm{T}h, \mathrm{T}g, \mathrm{T}i\rangle\neq\langle \mathrm{T}h, \mathrm{T}i, \mathrm{T}g\rangle$.

We cannot hope that another simple "rule of amalgamation" applies, either. Suppose that in the above, $U=V$, but all other objects listed are distinct; so we have the previous example, still with non-standard amalgamation. But also, for $\beta' = f(U, g(X))$, $\beta'\Box_f\alpha=h(U, i(g(X)))$, and $\mathrm{S}(\beta'\Box_f\alpha)=\langle h, i, g\rangle\neq\mathrm{S}(\beta\Box_f\alpha)$, despite the fact that

$S(\beta')=S(\beta)$. That is, the source of a composite does not depend just on the sources (and targets) of the composed arrows, unlike in the ordinary, Lambek, multicategory.

There is a generalized notion of "multicategory" which allows for "non-standard" amalgamation. In this we have, as part of the structure, so-called *amalgamating maps* $\psi=\psi[\beta, \alpha, p]$, $\varphi=\varphi[\beta, \alpha, p]$:

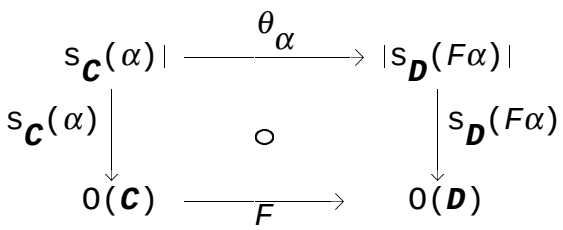
$$s(\beta)\backslash p \xrightarrow{\ \psi\ } s(\beta\circ_p\alpha) \xleftarrow{\ \varphi\ } s(\alpha)$$

associated with any meaningful composition $(\beta, \alpha, p)\longmapsto \beta\circ_p\alpha$, which puts together the source of $\beta\circ_p\alpha$ in a specific, but *a priori* undetermined, way from the source of β (take away the symbol at place p) and the source of α . The notation abbreviates the following: ψ is a map from the set $|s(\beta)|-\{p\}$ to the set $|s(\beta\circ_p\alpha)|$ (where $|s|=\mathrm{dom}(s)$, and $s\backslash p=s\restriction(|s|-\{p\})$) such that

$$\begin{array}{ccc} |s(\beta)|-\{p\} & \xrightarrow{\ \psi\ } & |s(\beta\circ_p\alpha)| \\ & {\scriptstyle s(\beta)\backslash p}\searrow \quad \circlearrowleft \quad \swarrow{\scriptstyle s(\beta\circ_p\alpha)} & \\ & o & \end{array} \quad ,$$

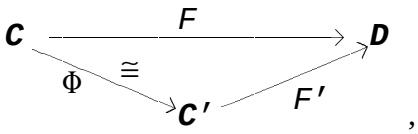
and similarly for φ . In the standard case, the amalgamating maps correspond to the fact that in $s(\beta\circ_p\alpha)$, " $s\alpha$ is inserted in $s\beta$ in the place p ". In the generalized concept, there are coherence conditions on the amalgamating maps, one for each of the four laws: the unit laws, the associative law, and the commutative law. The above-described structure $\boldsymbol{D}$ is a multicategory in the generalized sense (in comparing this part with the official definition of section 2, and the definition of $\boldsymbol{D}$ in section 5 , note that the concept being described here is a *1-level* multicategory as opposed the more general 2-level version given in those sections; we will comment on the reason for the 2-level version later in the introduction).

The reason for the general concept of multicategory and for the particular multicategory $\boldsymbol{D}$ is to provide a concept under which $d:P_{k+1}\longrightarrow P_k$ becomes a morphism of multicategories. A *morphism* $F:\boldsymbol{C}\to\boldsymbol{D}$ *of multicategories* maps objects to objects, arrows to arrows, but, instead of being compatible strictly with the source-assignments, it has a system of transition isomorphisms $\theta_\alpha: |s_{\boldsymbol{C}}(\alpha)| \xrightarrow{\ \cong\ } |s_{\boldsymbol{D}}(F\alpha)|$ ($\alpha\in A(\boldsymbol{C})$) such that



F is to preserve placed composition; in formulating this, the transition isomorphisms play a role: given that $\beta\circ_p\alpha$ is well-formed in $\boldsymbol{C}$, $F\beta\circ_q F\alpha$ for $q=\theta_\beta(p)$ is well-formed in $\boldsymbol{D}$; we require that $F(\beta\circ_p\alpha)=F\beta\circ_q F\alpha$. It is also required that the θ_α be compatible with the amalgamating maps.

There is a trade-off between amalgamating maps and transition isomorphisms. Given any morphism $F:\boldsymbol{C}\to\boldsymbol{D}$ of multicategories, there is a factorization of F,



in which Φ is an isomorphism, and in fact, it is an identity on both objects and arrows, and F' is *strict*, its transition isomorphisms are all identities. In other words, by changing the domain to an isomorphic copy, albeit with "twisted" amalgamating maps, it is possible to turn a morphism into a strict one.

1.6. Constructing higher dimensional cells

We are ready to summarize the construction of higher dimensional cells. Assuming that we have a set C_k of k-cells for $k=0, 1, \ldots, n$, and we have defined k-pd's for the same k's, with domain and codomain maps $\mathrm{d}:P_{k+1}\to P_k$, $\mathrm{c}:P_{k+1}\to C_k$, we introduce $(n+1)$-cells $a\in C_{n+1}$ by declaring each $\mathrm{d}a=\mathrm{d}_{n+1}(a)$ and $\mathrm{c}a=\mathrm{c}_{n+1}(a)$ to be a specific n-pd $\alpha=\mathrm{d}a$, resp. n-cell $b=\mathrm{c}a$ *such that* $\mathrm{d}\alpha=\mathrm{d}b$, $\mathrm{c}\alpha=\mathrm{c}b$, that is,

$$\mathrm{dd}a=\mathrm{dc}a\,,\ \mathrm{cd}a=\mathrm{cc}a\,. \tag{9}$$

We let $\mathbf{D}_n$ be the multicategory of function-replacement based on $\mathbf{C}_n$, the free multicategory with arrows the n-pd's, and $\mathbf{C}^0_{n+1}$ be the free multicategory with standard amalgamation, and with objects the n-cells, and generating arrows the $(n+1)$-cells just declared; in other words, $\mathbf{C}^0_{n+1}=\mathcal{F}(\mathcal{L})$ where $\mathtt{O}(\mathcal{L})=C_n$, $\mathtt{L}(\mathcal{L})=C_{n+1}$, and in which $\mathtt{s}_{\mathcal{L}}(a)=\langle \mathtt{d}a\rangle$, $\mathtt{t}_{\mathcal{L}}(a)=\mathtt{c}a$ ($a\in C_{n+1}$). P_{n+1} is the set of arrows in $\mathbf{C}^0_{n+1}$. The main step in the definition is to define $\mathtt{d}^0=\mathtt{d}^0_{n+1}:\mathbf{C}^0_{n+1}\longrightarrow\mathbf{D}_n$ by the freeness of $\mathbf{C}^0_{n+1}$ as to extend the determination of $\mathtt{d}$ on C_{n+1}. For this, it is crucial that $\mathbf{C}^0_{n+1}$, although it is defined as a Lambek multicategory, it remains free on $\mathcal{L}$ in the larger category of all multicategories with possibly non-standard amalgamation. Finally, we alter $\mathbf{C}^0_{n+1}$ to the *isomorphic copy* $\mathbf{C}_{n+1}$ by "twisting" the amalgamation maps to ensure that $\mathtt{d}:\mathbf{C}_{n+1}\longrightarrow\mathbf{D}_n$ is strict. As a result, we get the *main formula* saying that

$$\mathtt{d}(\beta\circ_p\alpha)=(\mathtt{d}\beta)\square_p(\mathtt{d}\alpha) \qquad (10)$$

every time $\beta\circ_p\alpha$ is a meaningful composition in $\mathbf{C}_{n+1}$.

Let us see the effect of the above general procedure for some particular 3-cells and 3-pd's. In what follows, U, V, W, ... denote 1-cells, f, g, h, ... 2-cells, u, v, 3-cells; Greek letters are used to denote pd's of various dimensions.

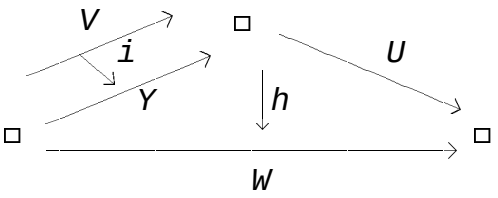
We adopt a single 0-cell that we indicate by $\square$; the 1-cells U, V, W, X, Y are all like $\square\longrightarrow\square$. The 2-cells f, g, h, i are as in (8). We add

$$k:\langle X,V\rangle\longrightarrow W,\quad \ell:\langle U,X\rangle\longrightarrow W.$$

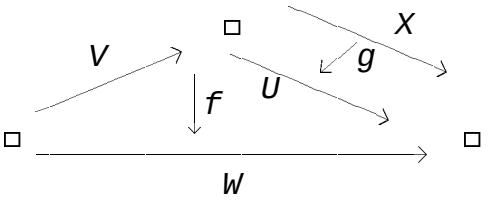
We are assuming that $U=V$, but all other 1-cells denoted differently are distinct.

Consider the 2-pd's $\alpha=h(U,i(U))$ and $\beta=f(g(X),U)$:

α :

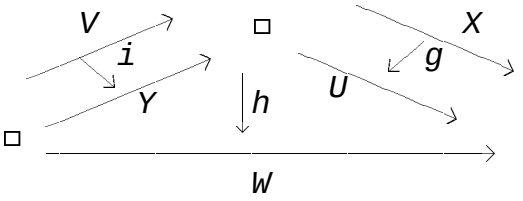


β :



We have $\beta \square_1 \alpha = h(g(X), i(U))$:

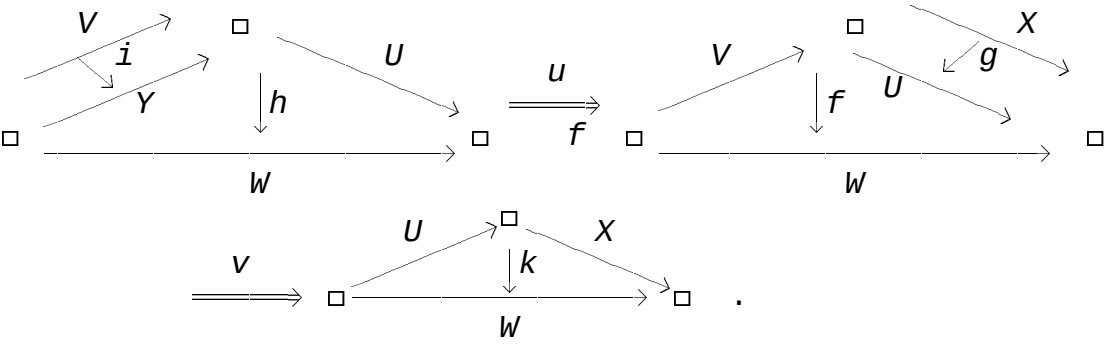
$\beta \square_1 \alpha$:



.

We introduce the 3-cells u and v by declaring $\mathrm{d}u=\alpha$, $\mathrm{c}u=f$ and $\mathrm{d}v=\beta$, $\mathrm{c}v=k$; the globularity conditions (9) are satisfied. We let

$$\psi = v \circ_1 u = v(u(h, i), g) :$$



.

We have

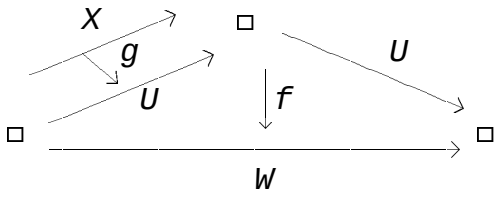
$$\mathrm{d}\psi = \mathrm{d}(v \circ_1 u) = (\mathrm{d}v) \square_1 (\mathrm{d}u) = \beta \square_1 \alpha ;$$

so

$$s_{\boldsymbol{C}_3}(\psi) = \langle \mathrm{d}\psi \rangle = \langle h, g, i \rangle \ .$$

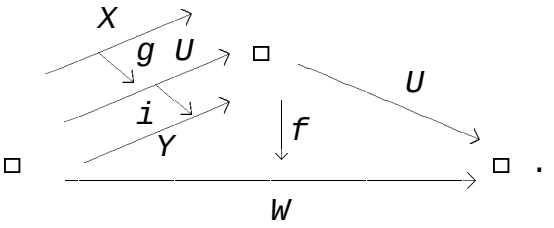
Now, look at

$$\beta' = f(U, g(X)) :$$



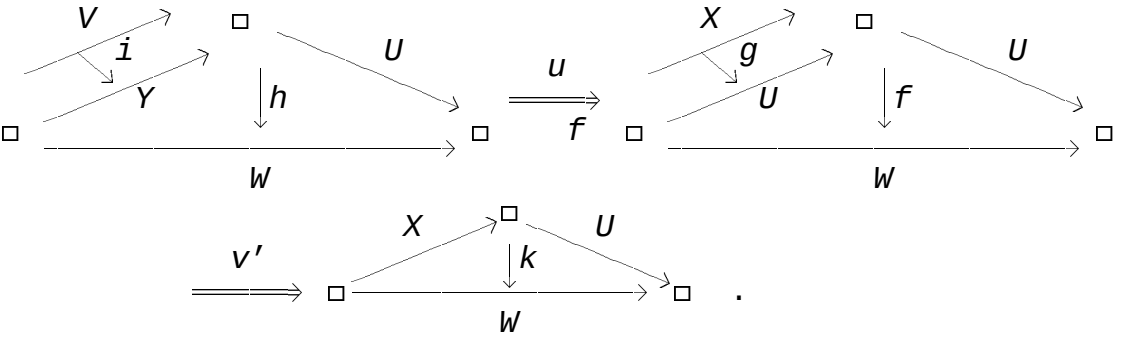
and

$$\beta' \square_1 \alpha = \ h(U, i(g(X))) :$$



We let $v' \in C_3$ with $\mathrm{d}v' = \beta'$, $\mathrm{c}v' = k$ ($\mathrm{d}\beta' = \mathrm{d}k$, $\mathrm{c}\beta' = \mathrm{c}k$ hold), and

$$\psi' = v' \circ_1 u = v'(u(h, i), g) :$$



We have

$$s_{\boldsymbol{C}_3}(\psi') = \langle \mathrm{d}\psi' \rangle = \langle h, i\, g \rangle \ .$$

We just have to get used to the fact that

$$s_{\boldsymbol{C}_3}(v(u(h, i), g)) = \langle h, g, i \rangle ,$$

and

$$s_{\boldsymbol{C}_3}(v'(u(h, i), g)) = \langle h, i, g \rangle$$

at the same time. Of course, this does not look so surprising if we look at the *full* representations of the two 3-pd's $\psi = v(u(h, i), g)$ and $\psi' = v'(u(h, i), g)$, which are different "geometrically".

1.7 Introducing two levels of objects

Some remarks concerning the "2-leveled" version for the notion of multicategory, for whose definition we refer to section 2. This is introduced purely for technical convenience. The 2-leveled notion packs more structure into the multicategory $\boldsymbol{D}$ of function-replacement, structure that is already there "naturally". For instance, instead of having the source of α as $s_{\boldsymbol{D}}(\alpha) = \langle T(f_1), \ldots, T(f_n) \rangle$, we have it, in the 2-leveled version of $\boldsymbol{D}$, as $s_{\boldsymbol{D}}(\alpha) = \langle f_1, \ldots, f_n \rangle = \langle \alpha \rangle$. The effect is to *restrict the scope* of the composition operation $\Box$; composition in the 2-leveled version remains the same as in the 1-leveled version, but it is defined for a subset of the domain of the 1-leveled composition. For $\gamma, \delta \in \boldsymbol{D}$, the composite $\delta \Box_p \gamma$ is meaningful, in the 2-leveled version, if and only if $p \in |\langle \delta \rangle|$, and for $f = \langle \delta \rangle(p)$, we have $df = d\gamma$ and $cf = c\gamma$. This is in fact the case exactly when the function-replacement composite is the *meaningful geometrically*. Under the 1-leveled version, the multicategory $\boldsymbol{D}$ has composites that cannot be realized geometrically in Euclidean space.

The 2-leveled concept helps technically. An example is the equality $d\alpha = \langle \alpha \rangle$ holding for all $\alpha \in \boldsymbol{C}_{n+1}$. This is immediate if d is defined by the freeness of $\boldsymbol{C}_{n+1}$ with respect to the 2-leveled version of "multicategory"; it would require additional arguments if we used the 1-leveled version.

1.8. Final remarks

Obviously, for any fixed n, n-graphs are the objects of a category of the form $\mathtt{Set}^{\mathtt{g}_n}$; here, $\mathtt{g}_n$ is the category whose shape in given in (1). It turns out that n-dimensional multitopic sets, with a natural notion of morphism, also form a category of the form $\mathtt{Set}^{\mathtt{mt}_n}$. In this case, the description of the exponent category $\mathtt{mt}_n$, the category of n-dimensional *multitopes*, is less easy to describe. In fact, there is, apparently, no other way of describing $\mathtt{mt}_n$ than by the same recursive process that serves defining multitopic sets in general. The objects of $\mathtt{mt}_n$ are the same as the pasting diagrams (elements of the P_n-component) in the *terminal* n-dimensional multitopic set, the one that has exactly one cell in each possible type (domain/codomain pair; in fact, here "domain" suffices; this description is an oversimplification, and neglects an inherent recursion). The arrows of $\mathtt{mt}_n$ are more difficult to explain. The definition of the $\mathtt{mt}_n$ and the proof of their connection to multitopic sets in general are given in section 7.

The fact just stated is the justification for the name "multitopic set". It is a similar construction to "simplicial set" , with "simplices" in the background, and also to "opetopic set" of [B/D3], based on "opetopes", in which *operads*, the basic abstract concept for [B/D3], are referred to. We copied and modified "opetope" and "opetopic set" of [B/D3], bearing in mind multicategories as the basic abstract concept, replacing operads.

We note that "higher dimensional (or: n-dimensional) multicategory", a term that may seem at first to be the appropriate one for our concept of multitopic set, is in fact incorrect and misleading. "Higher dimensional multicategory" would rightly be expected to generalize "multicategory"; however, in our multitopic sets only special multicategories figure, namely, the free ones, and another particular kind, the multicategories of function replacement, closely tied to the free multicategories. For multitopic sets, *particular* multicategories are used as a tool to describe a specific geometric arrangement, that of cells of various dimensions fitting together in pasting diagrams. Of course, this is similar to the use of operads in [B/D3].