Rules of (extended) natural deduction

Abbreviations: $\Psi :=: \Psi_1, \ldots, \Psi_k$; $\Psi' :=: \Psi'_1, \ldots, \Psi'_\ell$. ($k=0$, $\ell=0$ possible)

Structural rules:

I: $$\frac{}{\Phi \vdash \Phi}$$; **W:** $$\frac{\Psi \vdash \Phi}{\Psi, \Gamma \vdash \Phi}$$;

Int: $$\frac{\Psi, \Gamma, \Lambda, \Psi' \vdash \Phi}{\Psi, \Lambda, \Gamma, \Psi' \vdash \Phi}$$; **Con:** $$\frac{\Psi, \Gamma, \Gamma \vdash \Phi}{\Psi, \Gamma \vdash \Phi}$$; **Cut:** $$\frac{\Psi \vdash \Lambda \qquad \Psi, \Lambda \vdash \Phi}{\Psi \vdash \Phi}$$

T: $$\frac{\Psi \vdash \Phi_1 \qquad \Psi \vdash \Phi_2 \qquad \ldots \qquad \Psi \vdash \Phi_k}{\Psi \vdash \Phi}$$

Proviso: Φ is a tautological (Boolean) consequence of $\Phi_1, \ldots, \Phi_k$.

D: $$\frac{\Psi, \Gamma \vdash \Phi}{\Psi \vdash \Gamma \to \Phi}$$

C: $$\frac{\Psi, \Phi \vdash \Gamma \qquad \Psi, \Phi \vdash \neg\Gamma}{\Psi \vdash \neg\Phi}$$

AC: $$\frac{\Psi, \Gamma \vdash \Phi \qquad \Psi, \Lambda \vdash \Phi}{\Psi, \Gamma \vee \Lambda \vdash \Phi}$$

US: $$\frac{\Psi \vdash \forall x \Phi}{\Psi \vdash \Phi[t/x]}$$ **EG:** $$\frac{\Psi \vdash \Phi[t/x]}{\Psi \vdash \exists x \Phi}$$

Proviso for US and EG **:** the substitution $\Phi[t/x]$ is legal

UG: $$\frac{\Psi \vdash \Phi}{\Psi \vdash \forall x \Phi}$$ **Proviso:** the variable x is not free in Ψ .

ES: $$\frac{\Psi, \Gamma \vdash \Phi}{\Psi, \exists x \Gamma \vdash \Phi}$$ **Proviso:** the variable x is not free in Ψ , Φ .

CBV: $$\frac{\Psi \vdash \Phi}{\Psi \vdash \Phi'}$$: Φ' is obtained from Φ by a legal change of bound variables

E: $$\frac{}{\emptyset \vdash t=t}$$; $$\frac{\Psi \vdash t_1=t_2 \qquad \Psi \vdash \Phi[t_1/x]}{\Psi \vdash \Phi[t_2/x]}$$

$$\frac{\Psi \vdash t_1=t_2 \qquad \Psi \vdash \Phi[t_2/x]}{\Psi \vdash \Phi[t_1/x]}$$