

The invariance theorem

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• Data:

$$\Delta = \wedge \neg \exists$$

(Boolean)

$$\text{or } \wedge \vee \rightarrow \exists \forall$$

(Heyting)

$$\begin{array}{c} \mathbb{E} \\ \downarrow \mathcal{E} \\ \mathbb{B} \end{array}$$

$$\begin{array}{c} \mathbb{F}_1 \\ \downarrow \mathcal{D}_1 \\ \mathbb{C}_1 \end{array}$$

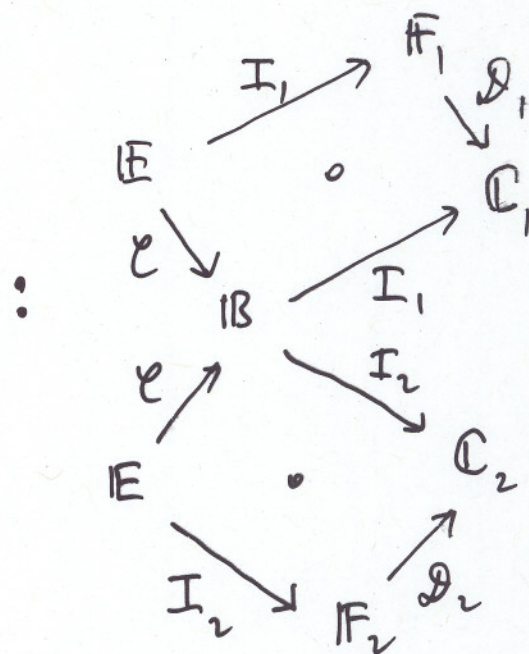
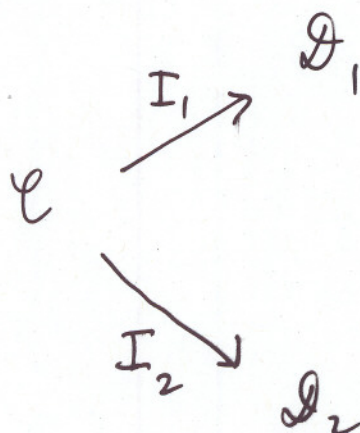
$$\begin{array}{c} \mathbb{F}_2 \\ \downarrow \mathcal{D}_2 \\ \mathbb{C}_2 \end{array}$$

$$Q \subset \text{Arr}(\mathbb{B})$$

Δ -Q-fibrations

($\mathcal{D}_1, \mathcal{D}_2$: full)

interpretations:

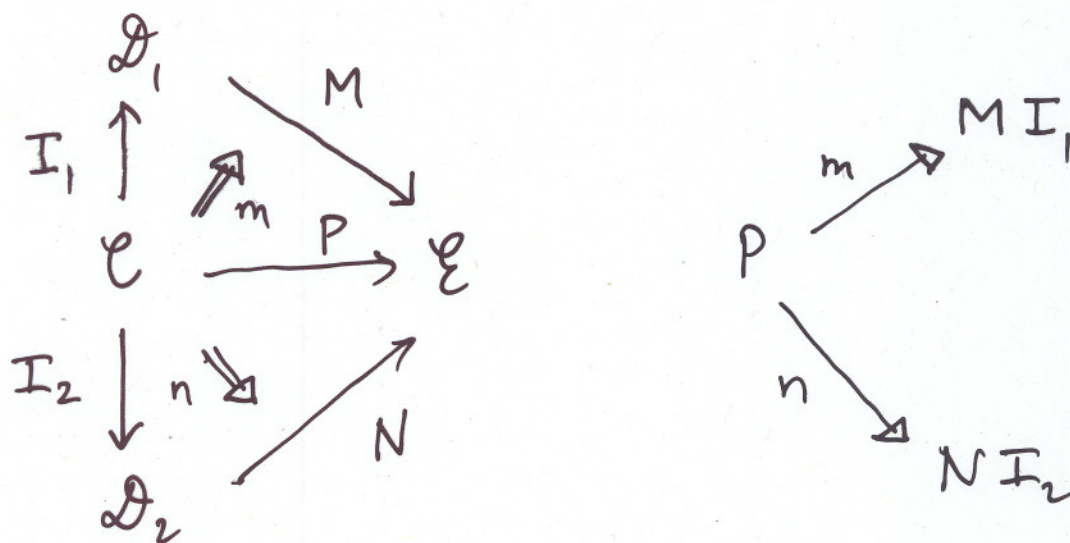


- 'Generic Q -equivalence of a $\mathcal{D}_1$ -model and a $\mathcal{D}_2$ -model':

Definition. • let $\mathcal{E}$ be a (further) full Δ -fibration.

An $\mathcal{E}$ -cocone (a cocone with vertex $\mathcal{E}$ and base $\mathcal{D}_1 \xleftarrow{I_1} \mathcal{C} \xrightarrow{I_2} \mathcal{D}_2$) is:

(M, N, P, m, n) s.t.:



- A morphism of $\mathcal{E}$ -cocones:

$$(M, N, P, m, n) \xrightarrow{(f, g, h)} (M', N', P', m', n') : \text{Compatibility:}$$

$$\begin{array}{ccccc}
 & & MI_1 & \xrightarrow{f} & M'I_1 \\
 & \nearrow m & & \cong & \nwarrow m' \\
 P & \xrightarrow{\quad} & & \circ & P' \\
 & \searrow n & \xrightarrow{h} & \circ & \nwarrow n' \\
 & & NI_2 & \xrightarrow{g} & N'I_2
 \end{array}$$

- $\text{Cocone}(\mathcal{E})$: category of $\mathcal{E}$ -cocones and their morphisms.

Definition Let $\mathcal{D}, \mathcal{E}$ be full Δ -fibr.

$\Delta^{\cong}(\mathcal{D}, \mathcal{E})$ is the $\{\text{groupoid}\}$ of Δ -maps and their isomorphisms:

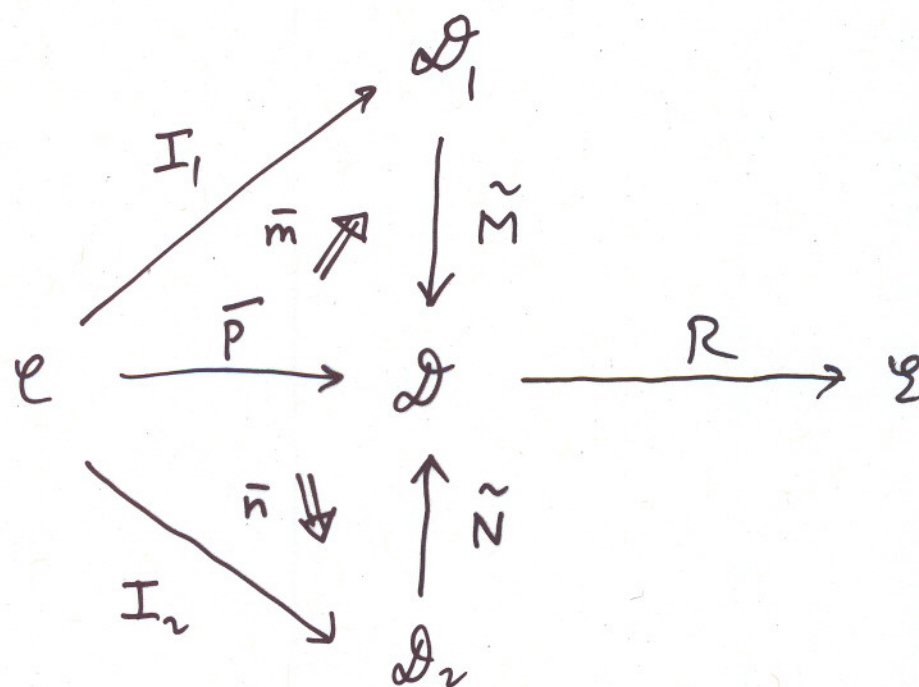
$$\begin{array}{ccc}
 [\mathcal{D}] & \xrightarrow[\substack{h \downarrow \\ N}]{M} & [\mathcal{E}] \\
 \mathcal{D} \downarrow & & \downarrow \mathcal{E} \\
 |\mathcal{D}| & \xrightarrow[\substack{h \downarrow \\ N}]{M} & |\mathcal{E}|
 \end{array}
 \quad \text{---} \quad \text{---} \quad \text{---} \quad h: M \rightarrow N$$

Given $\mathcal{D}$ -cocone $\gamma = (\tilde{M}, \tilde{N}, \bar{P}, \bar{m}, \bar{n})$

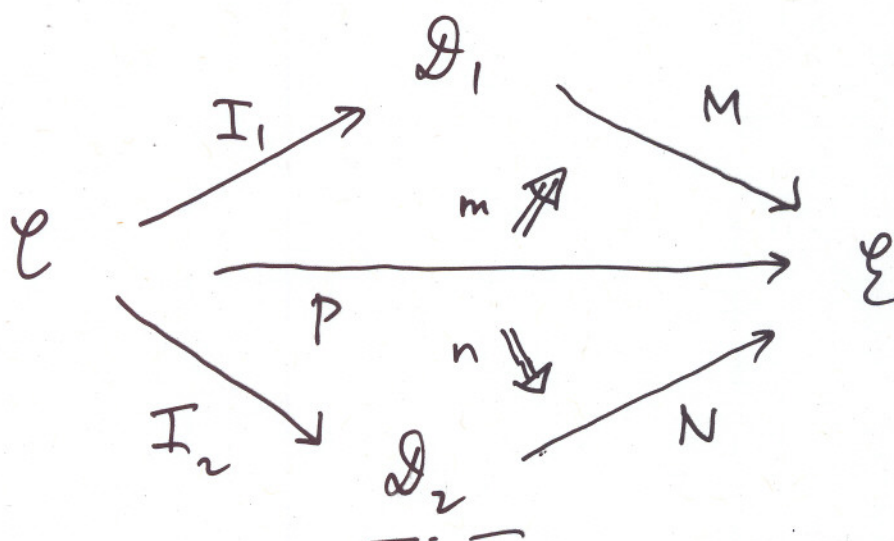
and $R: \mathcal{D} \rightarrow \mathcal{E} \in \Delta(\mathcal{D}, \mathcal{E})$

we have

$$(M, N, P, m, n) = (R\tilde{M}, R\tilde{N}, R\bar{P}, R\bar{m}, R\bar{n}) \in \text{Cocone}(\mathcal{E}) :$$



γ^*



$$\gamma^*: \Delta^{\sim}(Q, Q) \longrightarrow \text{cone}(Q)$$

Definition (Generic cone; generic Q -equivalence of a $\mathcal{D}_1$ -model and a $\mathcal{D}_2$ -model)

$(\mathcal{D}, \gamma)$ is generic if $\mathcal{D}$ is a Δ -fib, $\gamma \in \text{Cocone}(\mathcal{D})$ and

$$\gamma^*: \Delta^{\cong}(\mathcal{D}, \mathcal{E}) \longrightarrow \text{Cocone}(\mathcal{E})$$

is an equivalence of categories for all Δ -fibs $\mathcal{E}$.

Proposition

$(\mathcal{D} = \mathcal{D}_1 \underset{\mathcal{E}}{\overset{\approx Q}{+}} \mathcal{D}_2, \gamma)$ generic exists & is unique up to equivalence

• Interpolation property

Definition A $\mathcal{D}$ -cocone (M, N, P, m, n) has the interpolation property if:

$$\forall: X \in \mathcal{B}, \quad \Psi_1 \in \mathcal{D}_1^{I_1 X}, \quad \Psi_2 \in \mathcal{D}_2^{I_2 X}$$

$$\begin{array}{ccccc}
 & m_X^* M \Psi_1 & \leq_{PX} & n_X^* N \Psi_2 & \\
 & \searrow \omega & & \swarrow \omega & \\
 M \Psi_1 & & & & N \Psi_2 \\
 \omega \downarrow & & & & \downarrow \omega \\
 & m_X & PX & n_X & \\
 & \swarrow & & \searrow & \\
 MI_1 X & & & & NI_2 X
 \end{array}$$

$$\exists: \Phi \in \mathcal{C}^X$$

$$\Psi_1 \leq_{I_1 X} I_1 \Phi, \quad I_2 \Phi \leq_{I_2 X} \Psi_2$$