$\boxed{\text{Alternative, but equivalent, definition of 'saturated anafunctor'}}$, due to J. Baez & J. Dolan (later than 'anafunctor')

$$F : \mathbb{X} \xrightarrow{\text{virt.}} \mathbb{A} \qquad (\text{"virtual functor"})$$

is

$$F : \mathbb{X}^{op} \times \mathbb{A} \longrightarrow \mathrm{Set} \qquad (\text{ord. functor})$$

such that, for all $X \in \mathrm{Ob}(\mathbb{X})$

$$F(X, -) : \mathbb{A} \longrightarrow \mathrm{Set}$$

is representable.

Let us write

$$a : X \xrightarrow[\odot]{} A$$

for $\qquad a \in F(X, A)$

We used dot, $\cdot$, to indicate a specification $s : X \xrightarrow[\cdot]{} A$. To obtain the original concept, define $\quad s : X \xrightarrow[\cdot]{} A \overset{\Longleftrightarrow}{\scriptstyle def}$

$$(A, \underbrace{s \in F(X, A)}_{\left(s : X \xrightarrow[\odot]{} A\right)}) \text{ is a representation of}$$

$F(X, -)$, i.e.,

$$\mathbb{A}(A, -) \xrightarrow{\cong} F(X, -)$$

$$1_A \in \mathbb{A}(A, A) \longmapsto s \in F(X, A)$$

Further notation:

commutes

$$\begin{tikzcd} X \arrow[r, "\odot", "a"'] \arrow[dr, "\odot", "b"'] & A \arrow[d] \arrow[dl, phantom, "\mathrm{O}"] \\ & B \end{tikzcd} \quad \underset{\text{def}}{\iff} \quad F(X, f)(a) = b$$

$$\begin{tikzcd} X \arrow[r, "\odot", "a"'] & A \\ Y \arrow[u, "g"] \arrow[ur, "\odot", "c"'] \arrow[u, phantom, "\mathrm{O}"'] & \end{tikzcd} \quad \underset{\text{def}}{\iff} \quad F(g, A)(a) = c$$

Then $s : X \xrightarrow[\text{'universal'}]{\cdot} A \iff \forall b : X \xrightarrow{\odot} B$

$\exists! f$

$$\begin{tikzcd} X \arrow[r, "\cdot", "s"'] \arrow[dr, "\odot", "b"'] & A \arrow[d, "f"] \arrow[dl, phantom, "\mathrm{O}"] \\ & B \end{tikzcd}$$

As if s 'had an inverse'

except that no arrows $X \longleftarrow A$

The composition

$$\begin{tikzcd} X \arrow[r, "\odot"] & A \arrow[d, "f"] \\ Y \arrow[u, "g"] & B \end{tikzcd}$$

is associative

Conversely, starting with the original formulation, from $F : \mathbb{X} \xrightarrow{\text{ana}} \mathbb{A}$ (sat.) can define arrows $X \xrightarrow{\sigma} A$ as triples (B, s, f) where $B \in \mathrm{Ob}(\mathbb{A})$, $s \in |F|(X, B)$, $f : B \to A$

(think of

$$\begin{tikzcd} & B \arrow[d, "f"] \\ X \arrow[ur, "\bullet", "s"'] \arrow[r, "\sigma", "{(B,s,f)}"'] \arrow[ur, phantom, shift right=3, "\circ"] & A \end{tikzcd}$$

)

when (B, s, f) and (C, t, g) are identified when:

$$\begin{tikzcd} & B \arrow[d, "f"] \arrow[dd, bend left=70, "{F_{s,t}(1_X)}"] \\ X \arrow[ur, "\bullet", "s"'] \arrow[r, shift left=2, "{(B,s,f)}"] \arrow[r, phantom, "\shortparallel"] \arrow[r, shift right=2, "{(C,t,g)}"'] \arrow[dr, "\bullet", "t"'] & A \arrow[d, phantom, shift left=6, "\circ"] \\ & C \arrow[u, "g"'] \end{tikzcd}$$

commutes