

Alternative, but equivalent, definition of
'saturated anafunctor', due to J. Baez & J. Dolan
(later than anafunctor)

$$F: \mathbb{X} \xrightarrow{\text{virt.}} \mathbb{A} \quad (\text{"virtual functor"})$$

is

$$F: \mathbb{X}^{\text{op}} \times \mathbb{A} \longrightarrow \text{Set} \quad (\text{ord. functor})$$

such that, for all $X \in \text{Ob}(\mathbb{X})$

$$F(X, -): \mathbb{A} \longrightarrow \text{Set}$$

is representable.

Let us write

$$a: X \xrightarrow{\circlearrowright} A$$

for $a \in F(X, A)$

We used dot, $\circ$, to indicate a specification
 $s: X \xrightarrow{\circ} A$. To obtain the original concept,

define $s: X \xrightarrow{\circ} A \xLeftrightarrow[\text{def}]{} (A, s \in F(X, A))$ is a representation of
 $(s: X \xrightarrow{\circ} A)$

$F(X, -)$, i.e.,

$$\mathbb{A}(A, -) \xrightarrow{\cong} F(X, -)$$

$$1_A \in \mathbb{A}(A, A) \longmapsto s \in F(X, A)$$

Further notation:

$$\begin{array}{ccc}
 X & \xrightarrow[\text{a}]{\textcircled{a}} & A \\
 \text{commuter} \swarrow & \text{O} & \downarrow \\
 & \text{b} & B
 \end{array}
 \quad \stackrel{\text{def}}{\Leftrightarrow} \quad F(X, f)(a) = b$$

$$\begin{array}{ccc}
 X & \xrightarrow[\text{a}]{\textcircled{a}} & A \\
 \uparrow \text{g} & \text{O} & \nearrow \\
 Y & & c
 \end{array}
 \quad \stackrel{\text{def}}{\Leftrightarrow} \quad F(g, A)(a) = c$$

$$\text{Then } s: X \xrightarrow[\text{'universal'}]{\cdot} A \quad \Leftrightarrow \quad \forall b: X \xrightarrow[\exists! f]{\textcircled{\cdot}} B$$

$$\begin{array}{ccc}
 X & \xrightarrow[\cdot]{s} & A \\
 \searrow & \text{O} & \downarrow f \\
 & b & B
 \end{array}$$

As if s 'had an inverse'
except that no arrows $X \leftarrow A$

The composition

$$\begin{array}{ccc}
 X & \xrightarrow{\textcircled{\cdot}} & A \\
 \uparrow g & & \downarrow f \\
 Y & & B
 \end{array}$$

is associative

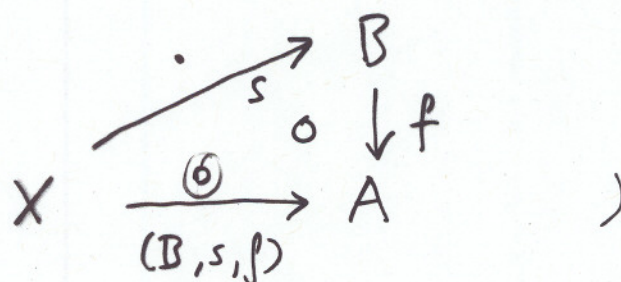
Conversely, starting with the original formulation, from $F : X \xrightarrow{\text{ana}} A$ (sat.)

can define arrows $X \xrightarrow{\odot} A$ as

triples (B, s, f)

where $B \in \text{Ob}(A)$, $s \in F(X, B)$, $f : B \rightarrow A$

(think of



when (B, s, f) and (C, t, g) are identified when:

