McGill University
Math 371B: Algebra IV
Assignment 1: due Friday January 30, 1998

Read Text: sections 2.9,2.10,2.11.

1. Let S be a compact subset of $\mathbb{R}$ and let $C(S)$ be the $\mathbb{R}$-algebra of continuous real-valued functions on S. For every $s \in S$ let $\epsilon_s : S \to \mathbb{R}$ be the mapping defined by $\epsilon_s(f) = f(s)$.
   (a) Show that ϵ_s is a homomorphism of $\mathbb{R}$-algebras whose kernel M_s is a maximal ideal of $C(S)$. The ideal M_s is called a point ideal of $C(S)$.
   (b) Show that every maximal ideal of $C(S)$ is a point ideal and that every $\mathbb{R}$-algebra homomorhism of $C(S)$ into $\mathbb{R}$ is equal to ϵ_s for some $s \in S$. **Hint:** Text, p. 111, #14.
2. (a) Text: p. 118, #5. If ϕ is the canonical mapping of R into RS^{-1} sending a to $a/1$, show also that for any homomorphism ψ_0 of R into a ring R' in which the elements of S are sent into invertible elements of R' there is a unique homomorphism ψ of RS^{-1} into R' such that $\psi_0 = \phi\psi$. (This property characterizes RS^{-1}.)
   (b) Let $B = AS^{-1}$, where A is a ring and S is a submonoid of A, and let $\phi : A \to B$ be the canonical homomorphism. If I is a prime ideal of A which does not meet S, show that $J = IB$ is a prime ideal of B such that $I = \phi^{-1}(J)$. Show also that every prime ideal J of B is equal to IB with $I = \phi^{-1}(J)$ and that the canonical mapping of $(A/I)(\phi_1(S))^{-1}$ into B/J is an isomorphism, where ϕ_1 is the canonical mapping of A onto A/I.
3. Text: p. 126, #1,2.
4. Text: p. 133, #2.
5. Text: p. 133, #3.
6. Text: p. 133, #4,5.
7. Text: p. 133, #9.
8. Text: p. 133, #10.
9. Text: p. 133, #11.
10. Text: p. 133, #14.