McGill University
Math 370A: Algebra III
Midterm Test (Part 1): November 6, 1998

1. (a) State and prove Cayley's Theorem for groups.
   (b) Using (a), find a permutation representation of $C_2 \times C_2$ in S_4.
2. (a) State and prove the First Isomorphism Theorem for groups.
   (b) Using (a), show that $d\mathbb{Z}/n\mathbb{Z} \cong \mathbb{Z}/\frac{n}{d}\mathbb{Z}$ if $d|n$.
3. (a) If A finite group G acts on a finite set X, show that the cardinality of the orbit containing $x \in X$ is the index in G of the stabilizer of x.
   (b) Using (a), for the action of a group on itself via inner automorphisms, prove that a finite p-group has a non-trivial center.
4. (a) Find the Sylow subgroups of A_4.
   (b) One of the groups $C_3 \times C_4$, $C_3 \rtimes C_4$, $(C_2 \times C_2) \rtimes C_3$, $S_3 \times C_2$ is isomorphic to A_4. Which is it? Justify your assertion.