McGill University
Math 325A: Differential Equations
Test 1

1. Solve the initial value problem

$$\frac{dy}{dx} = \frac{-1}{x + e^y}, \quad y(0) = 0.$$

   What is the interval of definition of your solution?

2. Find the general solution of the differential equation

$$x\frac{dy}{dx} = y^2 - y.$$

   Use your result to show that, given x_0, y_0, there is a unique solution with $y(x_0) = y_0$ if $x_0 \neq 0$. What happens if $x_0 = 0$?

3. A brine solution flows at a constant rate of 6 L/min into a large tank which initially holds 50 liters of brine solution in which are dissolved 5 kg of salt. The solution inside the tank is kept well stirred and flows out of the tank at the same rate. If the concentration of the salt in the brine entering the tank is 0.5 kg/L, determine the mass of salt in the tank at any time t. When will the concentation of salt in the tank reach 0.3 kg/L?

4. Consider the initial value problem $y' = x^2 + y^2, \ y(0) = 0$.

   (a) Show how the fundamental existence and uniqueness theorem can be used to show that there is a unique solution $y(x)$ on the interval $|x| \leq 1/2$. (Hint: consider the rectangle $|x| \leq 1, |y| \leq 1$.)

   (b) Find the first four terms $a_0 + a_1 x + a_2 x^2 + a_3 x_3$ of the Taylor series expansion of $y(x)$ about $x = 0$.

   (c) Find the second Picard interation $y_2(x)$. How good an approximation is it to $y(x)$? What can you say about $y_n(x)$, the n-th Picard interation?