McGill University
Math 325A: Differential Equations
Test 1 Solutions

1. Option B is correct by the fundamental existence and uniqueness theorem for linear differential equations. Option A is not true in general, for example, $y = 0$ is not a solution of $y'' = 1$. Option C is not true since $y_1 = x^2/2$ and $y_2 = x^2/2 + 1$ are solutions of $y'' = 1$ but $y_1 - y_2 = 1$ is not. Option D is not true since $y_1 = y_2 = x^2/2$ are solutions of $y'' = 1$ but $y_1 + y_2 = x^2$ is not.

2. Option A is not true since $6 = h(0) \neq -53f(0) + 27g(0) = 5$. Option C is not true since $6 = h(0) \neq -53f(0) + 27g(0) = 7$. Option D is not true in general even though $h(0) = -51f(0) + 26g(0)$ and $h'(0) = -51f'(0) + 26g'(0)$. For example, $f(x) = 32 + x$, $g(x) = 63 + 2x$, $h(x) = -51f(x) + 26g(x) + x^2$ are solutions of $y''' = 0$ which satisfy the given initial conditions but $h \neq -51f + 26g$ even though $h(0) = -51f(0) + 26g(0)$ and $h'(0) = -51f'(0) + 26g'(0)$. Hence option B is correct.

3. If y is a particular solution, we have $y \in \ker((D-1)^4(D+2)^2(D-2))$ and $y \notin \ker((D-1)^3(D+2)^2(D-2))$. The only option where this is possible is A.

4. IF $c \neq 1$, the solutions are of the form
$$Ae^{-7t} + Bte^{-7t} + C\sin(t) + E\cos(t)$$
which are bounded as $t \to +\infty$. If $c = 1$ the solutions are of the form
$$Ae^{-7t} + Bte^{-7t} + C\sin(t) + E\cos(t) + Ft\sin(t) + Gt\cos(t)$$
with F, G not both zero. Such a function is unbounded as $t \to +\infty$. Hence the only possible option is C.

5. The trace of the coefficient matrix is $c - 5$ and the determinant is $-(c+1)(c+6)$ which is ≥ 0 only when $-6 \leq c \leq -1$. Since the trace is less than zero in this range the solutions are bounded as $t \to +\infty$ precisely in this range. Hence option A is the correct one.