McGill University
Math 325A: Differential Equations
Assignment 3B Solutions

1. Let x be the distance travelled by the snow plow in t hours with $t = 0$ at 8 AM. Then if it started snowing at $t = -b$ we have
$$\frac{dx}{dt} = \frac{a}{t+b}.$$
The solution of this DE is $x = a\ln(t+b) + c$. Since $x(0) = 0$, $x(3) = 2$, $x(5) = 3$, we have $a\ln b + c = 0$, $a\ln(3+b) + c = 2$, $a\ln(5+b) + c = 3$ from which
$$a\ln\frac{3+b}{b} = 2, \quad a\ln\frac{5+b}{3+b} = 1.$$
Hence $(3+b)/b = (5+b)^2/(3+b)^2$ from which $b^2 - 2b - 27 = 0$. The positive root of this equation is $b = 1 + 2\sqrt{7} \approx 6.29$ hours. Hence it started snowing at $1:42:36$ AM.

2. The DE $y^3 + 4ye^x + (2e^x + 3y^2)y' = 0$ has an integrating factor $\mu = e^x$. The solution in implicit form is $2e^{2x}y + y^3e^x = C$. There is a unique solution with $y(x_0) = y_0$ for any x_0, y_0 by the fundamental existence and uniqueness theorem since the coefficient of y' in the DE is never zero and hence
$$f(x,y) = \frac{-y^3 - 4ye^x}{2e^x + 3y^2}$$
and its partial derivative f_y are continously differentiable on $\mathbb{R}^2$.

Alternately, since the partial derivative of $y^3e^x + 2ye^{2x}$ with respect to y is never zero, the implicit function theorem guarantees the existence of a unique function $y = y(x)$, with $y(x_0) = y_0$ and defined in some neighborhood of x_0, which satisfies the given DE.