McGill University
Math 325A: Differential Equations
Assignment 7: due Thursday, November 23, 2000

1. (a) Compute the Laplace transforms of the functions

$$t^2 \sin(t), \quad t^2 \cos(t).$$

   (b) Find the inverse Laplace tranforms of the functions

$$\frac{s}{(s^2+1)^3}, \quad \frac{1}{(s^2+1)^3}.$$

2. Using Laplace transforms, solve the initial value problem

$$y^{iv} - y = \sin(t), \quad y(0) = y'(0) = 1, y''(0) = y'''(0) = -1.$$

3. Using Laplace transforms, solve the system

$$\begin{aligned} \frac{dx}{dt} &= -2x + 3y, \\ \frac{dy}{dt} &= x - y \end{aligned}$$

   with the initial conditions $x(0) = 1, y(0) = -1$.

4. Using Laplace transforms, solve the initial value problem

$$y'' + 3y' + 2y = f(t), \quad y(0) = y'(0) = 0,$$

   where

$$f(t) = \begin{cases} 1, & 0 \le t < 1, \\ -1, & 1 \le t < \pi, \\ \sin(t), & \pi \le t. \end{cases}$$