

McGill University  
Math 325A: Differential Equations  
Assignment 4B: due Tuesday, October 24, 2000

1. (a) Show that the differential equation  $M + Ny' = 0$  has an integrating factor which is a function of  $z = x + y$  only if and only if

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M - N}$$

is a function of  $z$  only.

- (b) Use this to solve the differential equation

$$x^2 + 2xy - y^2 + (y^2 + 2xy - x^2)y' = 0.$$

2. Solve the differential equations

(a)  $xy'' = y' + x, \quad (x > 0);$

(b)  $y(y - 1)y'' + y'^2 = 0.$

3. Solve the differential equations

(a)  $y''' - 3y' + 2y = e^x;$

(b)  $y^{(iv)} - 2y''' + 5y'' - 8y' + 4y = \sin(x).$

4. Show that the functions  $\sin(x)$ ,  $\sin(2x)$ ,  $\sin(3x)$  are linearly independent. Find a homogeneous linear ODE having these functions as part of a basis for its solution space. Show that it is not possible to find such an ODE with these functions as a basis for its solution space.