McGill University
Math 319B: Partial Differential Equations

Class Test

Attempt All Questions
All Questions are of Equal Value

1. (a) Find the general solution $z = u(x, y)$ of the PDE

$$\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} = 1;$$

   (b) Find the equation of the characteristic curve passing through the point $(0, s, s^2)$;

   (c) Find the solution of the PDE with $u(0, y) = y^2$;

   (d) Show that the surface $z = u(x, y)$ found in (c) is the surface traced out by the family of curves found in (b).

2. (a) Prove that the general solution of the PDE

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = 0$$

   is $u(x, t) = f(x - ct) + g(x + ct)$;

   (b) Find the solution when $c = 2$, $u(x, 0) = x$, $\frac{\partial u}{\partial t}(x, 0) = e^x$.

3. Find the general solution of the PDE

$$\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial^2 u}{\partial t \partial x} + 2\frac{\partial u}{\partial x} - u = 0.$$