McGill University Math 319B: Partial Differential Equations Assignment 5: due Monday April 9, 2001

1. Solve the initial value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}, \quad x^2 + y^2 < 1, \ t > 0$$
$$u(\cos(\theta), \sin(\theta), t) = 0, \quad 0 \le \theta \le 2\pi, \ t \ge 0$$
$$u(x, y, 0) = f(r), \quad r = \sqrt{x^2 + y^2} \le 1$$

What is the physical problem described by these equations?

2. Find the eigenvalues and eigenfunctions for the regular Sturm-Liouville problem

$$u'' = -\lambda u, \quad 0 < x < 1,$$

 $u(0) = 0, \quad u'(1) = -u(1).$

Prove the orthogonality of the eigenfunctions.