

McGill University
Math 319B: Partial Differential Equations
Assignment 4: due Friday March 16, 2001

1. (a) Find the Fourier cosine series for the function $f(x) = 1 - x$ on the interval $0 \leq x \leq 1$;
(b) Using Fourier's Theorem, show that the series obtained in (a) converges for all real x . What is the function defined by this series? Graph it.
(c) Show that the series obtained by differentiating the above series term by term converges for all x . Describe the function defined by this series, What is the relation of this function to the function $f(x)$ given in (a).
2. (a) Find the Fourier sine series for the function $f(x) = 1 - x$ on the interval $0 \leq x \leq 1$;
(b) Using Fourier's Theorem, show that the series obtained in (a) converges for all real x . What is the function defined by this series? Graph it.
(c) For what values of x does the series obtained by differentiating the above series converge?
(d) What is the Fourier cosine series for the function $f'(x)$ on the interval $0 \leq x \leq 1$?
3. (a) Find the Fourier series of the function

$$f(x) = \begin{cases} 0 & -\pi \leq x \leq 0, \\ \sin(x) & 0 < x \leq \pi \end{cases}$$

and show that it converges to $f(x)$ for all x and to $f(x)$ for $-\pi \leq x \leq \pi$.

- (b) Show that the series obtained by differentiating the above series term by term converges for all x . What is the functions defined by this series? Graph it.
4. Integrate from 0 to x the Fourier series of the function in 2(a). What is the sum of the resulting series?