McGill University
Math 319B: Partial Differential Equations

Assignment 1: due Wednesday January 24, 2001

1. Which of the following operators L are linear?
   (a) $L(u) = \frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y}$;
   (b) $L(u) = \frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y}$;
   (c) $L(u) = \frac{\partial u}{\partial x} + (\frac{\partial u}{\partial y})^2$;
   (d) $L(u) = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + 1$.

   Justify your assertions.

2. (a) Find the general solution of the PDE
$$2\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} = 4.$$
   (b) Find the solution when $u(0, y) = y^2$ and sketch the surface $z = u(x, y)$.

3. (a) Find the characteristic curves of the PDE
$$\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} + u = e^{x+2y}.$$
   (b) Use (a) to find the general solution of the PDE.
   (c) Find the solution of the above PDE for which $u(x, 0) = 0$.

4. Solve the PDE
$$(1 + x^2)\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0.$$
   Find the solution with $u(0, y) = \sin(y)$.

5. Solve the PDE
$$\frac{\partial u}{\partial t} + 5t\frac{\partial u}{\partial x} = 3u$$
   subject to the initial condition $u(x, 0) = f(x)$.

6. (a) Find the general solution of the PDE
$$\frac{\partial u}{\partial x} - u^2\frac{\partial u}{\partial y} = 3u.$$
   (b) Find u in implicit form when the PDE is subject to the initial condition $u(0, y) = y$.