McGill University
Math 270A: Applied Linear Algebra
Test 1: due Thursday October 14, 1999

**Attempt all questions**
**No calculators allowed**
**Put your e-mail address on your exam booklet**

1. Let $V = \mathbb{R}^{[0,1]}$ be the real vector space of real-valued functions on the interval $0 \leq x \leq 1$.
   (a) Show that $W = \{f \in V \mid f(0) = f(1) = 0\}$ is a subspace of $V$.
   (b) Show that $W = \{f \in V \mid f(0)f(1) = 0\}$ is not a subspace of $V$ by exhibiting two functions $f, g \in W$ such that $f + g \notin W$.

2. Let $W$ be the subspace of $\mathbb{R}^{2\times 2}$ spanned by the matrices

$$A_1 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}.$$

   (a) Show that $W \neq \mathbb{R}^{2\times 2}$ by showing that $A_1, A_2, A_3, A_4$ are linearly dependent and quoting appropriate theorems; find a non-trivial dependence relation.
   (b) Find a basis of $W$ which is a subset of $\{A_1, A_2, A_3, A_4\}$. Justify your answer.

3. Given that the matrix $A = \begin{bmatrix} 7 & 8 & 22 & 8 & 15 \\ 8 & 7 & 23 & 7 & 15 \\ 6 & 9 & 21 & 9 & 15 \\ 2 & -2 & 2 & -2 & 0 \end{bmatrix}$

   is row equivalent to the matrix $B = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

   (a) find bases for the row space, column space and null space of $A$;
   (b) find the rank and nullity of $A$ and $A^T$. Justify your assertions.

4. Given that

$$W = \{x = (x_0, x_1, ..., x_n, ...) \in \mathbb{R}^\infty \mid x_{n+2} = 5x_{n+1} - 6x_n \text{ for all } n \geq 0\}$$

   is a 2-dimensional subspace of $\mathbb{R}^\infty$,

   (a) show that $u = (1, 2, ..., 2^n, ...)$, $v = (1, 3, ..., 3^n, ...)$ is a basis for $W$;
   (b) find the coordinate vector of $w = (1, 1, -1, ...) \in W$ with respect to the basis $u, v$.