McGill University
Math 270A: Applied Linear Algebra
Assignment 6: due Friday December 3 , 1999

1. Let $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$.

   (a) Find an orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix.

   (b) Find symmetric matrices Q_1, Q_2 such that $Q_1^2 = Q_1, Q_2^2 = Q_2, Q_1 + Q_2 = I, Q_1Q_2 = Q_2Q_1 = 0$ and $A = 2Q_1 + 5Q_2$.

   (c) Find a symmetric matrix B such that $B^2 = A$.

2. Let A be the matrix in question 1. Using 1(a),

   (a) find the maximum and minimum values of $Q(x, y, z) = 3(x^2 + y^2 + z^2) + 2(xy + xz + yz)$ on the unit sphere $x^2 + y^2 + z^2 = 1$.

   (b) evaluate
   $$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-Q(x,y,z)}\, dx\, dy\, dz.$$
   Hint: Make an appropriate change of variable and use the fact that
   $$\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}.$$

3. Show that the matrix $A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$ is a normal matrix. Find a unitary matrix U such that $U^{-1}AU$ is a diagonal matrix.

4. Find the singular value decomposition of the matrix $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and use it to find the least squares solution of the equations
$$\begin{aligned} x_1 - x_2 &= 1 \\ x_2 &= 1 \\ x_1 &= 1. \end{aligned}$$