McGill University
Math 270A: Applied Linear Algebra
Assignment 5: due Tuesday November 30 , 1999

1. Find the general solution of the system

$$x_{n+1} = .6x_n + .5y_n$$
$$y_{n+1} = -.1x_n + 1.2y_n$$

and show that $x_n - y_n$ tends to zero as $n$ tends to infinity. Find the solution with $x_0 = 10, y_0 = 30$.

2. Find the general solution of the linear recurrence equation $x_{n+2} - 1.8x_{n+1} + .77x_n = 0$ by expressing the solution set as the kernel of a polynomial in the left-shift operator $L$ and factoring this polynomial. Find the solution with $x_0 = 1, x_1 = 2$ and the solution with $x_0 = 2, x_1 = 1$

3. Find the general solution of the differential equation $f'''' - 10f''' + 37f'' - 60f' + 36f = 0$ by expressing the solution set as the kernel of a polynomial in the differentiation operator $D$ and factoring this polynomial. Hint: the roots of this polynomial are $2, 3$. Show that there is a unique solution with $f(0) = f'(0) = f''(0) = f'''(0) = 1$.

4. If $A = \begin{bmatrix} -3 & 1 & 1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{bmatrix}$ find the general solution of $\frac{dX}{dt} = AX$ by computing $e^{At}$. Show that $X(t) = [x(t), y(t), z(t)]^T$ converges to zero as $t$ tends to infinity. Show that there is a unique solution with $[x(0), y(0), z(0)] = [1, 2, 3]$.