McGill University
Math 270A: Applied Linear Algebra
Assignment 4: due Thursday November 11 , 1999

1. Find the (normalized) QR-decomposition of the matrix

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 1 & 4 & 6 \end{bmatrix}$$

and use it to find the least squares solution of the system

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 1 \\ x_2 + x_3 &= 1 \\ x_1 + 4x_2 + 6x_3 &= 1. \end{aligned}$$

2. Let $V = \mathbb{R}^{2\times 2}$ and let $T : V \to V$ be defined by $T(X) = CX - XC$, where $C \in \mathbb{R}^{2\times 2}$ is fixed.
   (a) Show that T is linear;
   (b) If C is not a scalar multiple of the identity, show that the kernel of T is at least 2-dimensional;
   (c) If $C = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, find bases for the kernel and image of T.
   (d) If C is as in (c), show that $T^3 = 16T$.

3. Let V be the vector space of polynomials $p(t) = a_0 + a_1 t + +a_2 t^2 + a_3 t^3$ with real coeficients a_i. Let $V \to V$ be defined by $T(p(t)) = t^2 p''(t) - 2p(t)$, where $p'(t)$ is the derivative of $p(t)$.
   (a) Show that T is linear;
   (b) Find bases for the kernel and image of T;
   (c) Find the matrix of T with respect to the basis $1, t, t^2, t^3$ and the matrix of T with respect to the basis $1+t, 1-t, t^2+t^3, t^2-t^3$; If A, B are respectively these two matrices, find a matrix P such that $P^{-1}AP = B$.

4. Let T be as in question 2(c) and let A be its matrix with respect to the basis

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

   (a) Find the charateristic polynomial of T and the eigenvalues of A;
   (b) Find a basis for each of the eigenspaces of A;
   (c) Find an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix.