McGill University
Math 270A: Applied Linear Algebra
Assignment 3: due Tuesday October 26 , 1999

1. If V is the real vector space $\mathbb{R}^X$ (X any set), show that its complexification $V_{\mathbb{C}}$ is isomorphic to $\mathbb{C}^X$. **Hint**: Show that any $f \in \mathbb{C}^X$ can be written in the form $f_1 + if_2$ with $f_1, f_2 \in \mathbb{R}^X$.

2. (a) Let S be a subset of $\mathbb{R}^2$ and let W_S be the subset of $\mathbb{R}^6$ consisting of those 6-tuples (a, b, c, d, e, f) such that $ax^2 + bxy + cy^2 + dx + ey + f = 0$ for all (x, y) in S. Show that W_S is a subspace of $\mathbb{R}^6$;
   (b) If $S = \{(1,1), (2,5), (3,0), (4,6)\}$, show that $\dim W_S = 2$;
   (c) Find a basis for W_S by taking products of suitable pairs of equations of lines passing through points of S;
   (d) Use the above to find the equation of the conic $ax^2 + bxy + cy^2 + dx + ey + f = 0$ which passes through the points of S and the point $(-1, 1)$.

3. Let V be the real inner product space of real-valued functions on the interval $[0, 1]$ with the inner product

$$< f, g >= \int_0^1 f(x)g(x)dx,$$

   let W be the subpace of polynomial functions of the form $f(x) = a+bx+cx^2$ with $a, b, c \in \mathbb{R}$ and let $T : \mathbb{R}^3 \to W$ is the function defined by $T((a, b, c)) = f$, where $f(x) = a + bx + cx^2$.
   (a) Show that T is an isomorphism of vector spaces.
   (b) Show that $< u, v >=< T(u), T(v) >$ is an inner product on $\mathbb{R}^3$ and find a formula for $||(a, b, c)||$ using this inner product.

4. Let W be the subspace of $\mathbb{R}^4$ spanned by the vectors

$$u_1 = (1, 2, 2, 1),\ u_2 = (1, 1, -1, -1),\ u_3 = (2, -1, -1, 2).$$

   Find the orthogonal projection of the vector $v = (a, b, c, d)$ on W and use this to find the least squares solution of the system

$$\begin{aligned} x + y + 2z &= a \\ 2x + y - z &= b \\ 2x - y - z &= c \\ x - y + 2z &= d. \end{aligned}$$

5. Using the notation of question 3, find the orthogonal projection of the function $f(x) = e^x$ on W. In what sense is this orthogonal projection the best approximation to $f(x) = e^x$ by a polynomial function of degree ≤ 3?