Final Examination MATH 262 December 12, 2005.

1. Find the interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-2)^{n^2}}{2n+1}$$

and determine whether the series converges absolutely or conditionally at the endpoints.

2. (a) Find the Maclaurin expansion for the function

$$F(x) = \int_0^x \sin(t^2)\, dt$$

and compute $F(1)$ to 2 decimal places.

(b) Find the Taylor series expansion of

$$f(x) = \frac{x-1}{1+(x-1)^2}$$

about $x = 1$ and use this to compute $f^{(9)}(1)$ and $f^{(10)}(1)$.

3. Find the solution of the differential equation

$$(1-x^2)y'' - xy' + 9y = 0$$

satisfying $y(0) = 0$, $y'(0) = 1$.

4. Given the curve $\mathbf{r} = t^2\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$,

(a) find the arc length of the curve from $t = 0$ to $t = 1$;

(b) find the unit tangent vector $\hat{\mathbf{T}}$, principal normal $\hat{\mathbf{N}}$, and binormal $\hat{\mathbf{B}}$ of the curve at the point $t = 1$;

(c) find the curvature κ and torsion τ of the curve at point $t = 1$.

5. Show that the function

$$f(x,y) = \begin{cases} \frac{ax+by^2}{\sqrt{x^2+y^2}} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

is continuous at $(0,0)$ if $a = 0$ but is not continuous at $(0,0)$ if $a \neq 0$.

6. (a) Find the tangent plane and normal line to the surface

$$xy + yz + xz^2 = 3$$

at the point $(1,1,1)$.

(b) Find the direction in which the function $f(x,y,z) = x^2 + xy + 2z^2$ has its maximum rate of increase at the point $P(1,1,1)$. What is the rate of increase at P in the direction from P to the point $Q(3,4,5)$.

7. A surface $z = f(x,y)$ has the parametric representation

$$x = u + v^2,\ y = u^2 - v^3,\ z = 2uv$$

in a neighbourhood of the point $(3,3,4)$ which corresponds to the point $(2,1)$ in the uv-plane. Find $\frac{\partial f}{\partial x}(3,3)$, $\frac{\partial f}{\partial y}(3,3)$, $\frac{\partial^2 f}{\partial x \partial y}(3,3)$.

8. Find and classify the critical points of the function $f(x,y) = 2x^3 - 6xy + 3y^2$. What is the Taylor polynomial of degree 2 in the Taylor expansion of $f(x,y)$ about each of the critical points?

9. Using the Lagrange multiplier method, find the maximum and minimum values of the function

$$f(x,y) = 2x + y$$

on the ellipse $x^2 + xy + y^2 = 1$.

McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATH 262

INTERMEDIATE CALCULUS

Examiner: Professor J. Labute
Associate Examiner: Professor J-J. Xu

Date: Monday, December 12, 2002
Time: 9:00 A.M. - 12 NOON

INSTRUCTIONS

Attempt all questions.
All questions are of equal value.
Justify all your statements.

This exam comprises the cover and 2 pages with 9 questions.