MATH 255: Midterm Test Solutions

The only thing that cannot be found in the notes is question 2(a).

2(a) By the Lebesgue integrability criterion, the function f has a point of continuity at some point $x_0 \in [a, b]$. Since $f(x_0) > 0$ and f is continuous at x_0, there is a closed interval $[c, d] \subseteq [a, b]$ of length > 0 such that $f(x) \geq m > 0$ on $[c, d]$. Since $f(x) > 0$ on $[a, b]$, we have $\int_a^b f(x)\,dx \geq \int_c^d f(x)\,dx \geq m(d - c) > 0$.