MATH 255: Assignment 7 Solutions

1. (a) $\sum_{k=1}^{n}(a_{k+1}-a_k)=a_{n+1}-a_1\to a-a_1$;
   (b) $\log(1-1/n^2) = \log((n^2-1)/n^2) = \log(n+1)-\log(n-1)-2\log n = b_{n+1}-b_n$ with $b_n=\log n-\log(n-1)=\log(n/n-1)\to 0$.
2. (a) $\sum_{n=1}^{\infty}(-1)^n/\sqrt{n}$ converges (conditionally) but $\sum_{n=1}^{\infty}1/n$ diverges;
   (b) If $\sum_{n=1}^{\infty}a_n$ is absolutely convergent, then $a_n\to 0 \implies |a_n|\le M$ and so $a_n^2\le M|a_n|$ which implies $\sum_{n=1}^{\infty}a_n^2$ converges.
3. (a) $na_{2n}<\sum_{k=n+1}^{2n}a_k\to 0$ and $na_{2n-1}<\sum_{k=n}^{2n-1}a_k\to 0$.
   (b) $\sum_{n=2}^{\infty}1/n\log n$
4. (a) $(\sqrt{n+1}-\sqrt{n})/\sqrt{n}=1/\sqrt{n}(\sqrt{n+1}+\sqrt{n})\sim 1/2n \implies$ divergence;
   (b) $(\sqrt{n+1}-\sqrt{n})/n=1/n(\sqrt{n+1}+\sqrt{n})\sim 1/2n^{3/2} \implies$ convergence.