MATH 255: Midterm Test

Time: 50 minutes February 19, 2003

Attempt all questions
All questions are of equal value

1. (a) Define what is meant for a function to be (i) Riemann-Stieltjes integrable, (ii) strictly Riemann-Stieltjes integrable with respect to a function α on $[a, b]$.

   (b) Using only the definition, show that a function f defined on $[0, 1.5]$ is Riemann-Stieltjes integrable with respect to the largest integer function $[x]$ if and only if f is left continuous at $x = 1$.

2. (a) If f is continuous on $[a, b]$ and α is increasing on $[a, b]$, show that f is strictly Riemann-Stieltjes integrable with respect to α.

   (b) Using (a), show that an increasing function on $[a, b]$ is Riemann integrable.

3. (a) Using the Cauchy Criterion for integrablity, show that a Riemann integrable function on $[a, b]$ is bounded.

   (b) If f is a non-zero function on $[a, b]$ and $f \geq 0$ on $[a, b]$, prove that $\int_a^b f(x)\,dx > 0$.

4. (a) If (f_n) is a sequence of functions on $S \subseteq \mathbb{R}$, define what it meant for (f_n) to converge uniformly to a function f on S. Give an example of a sequence of functions which converge pointwise on $[0, 1]$ but not uniformly. Prove all your assertions.

   (b) If $f_n \to f$ uniformly on $[a, b]$ and each f_n is continuous, show that f is continuous and that $\lim_{n\to\infty} \int_a^x f_n(t)\,dt = \int_a^x f(t)\,dt$ uniformly on $[a, b]$.