MATH 255: Assignment 8

(due Wednesday, March 19)

1. (a) Using a comparison test to a suitable p-series, determine the convergence or divergence of the following series

$$\text{(i)} \quad \sum_{n=2}^{\infty} \frac{1}{2^{\log n}}, \qquad \text{(ii)} \quad \sum_{n=2}^{\infty} \frac{1}{3^{\log n}}, \qquad \text{(iii)} \quad \sum_{n=2}^{\infty} \frac{1}{\log n^{\log n}}.$$

   (b) Using the estimate for the remainder, determine how many terms would be sufficient to compute the sum of each of the convergent series in (a) to within .001

2. (a) Using the ratio or root test, determine the convergence or divergence of the following series

$$\text{(i)} \quad \sum_{n=1}^{\infty} \frac{n!}{2^{e^n}}, \qquad \text{(ii)} \quad \sum_{n=1}^{\infty} \frac{n!}{n^n}, \qquad \text{(iii)} \quad \sum_{n=1}^{\infty} \frac{n^3 + 2^n}{e^n}.$$

   (b) Using the estimate for the remainder, determine how many terms would be sufficient to compute the sum of each of the convergent series in (a) to within .001

3. For which $p, q \in \mathbb{R}$ does $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(\log n)^p}{n^q}$ converge. For which p, q is the convergence absolute?

4. (a) If $s = \sum_{n=1}^{\infty} 1/n^2$, prove that

$$\text{(i)} \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = \frac{1}{2}s, \qquad \text{(ii)} \quad \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{3}{4}s.$$

   (b) Prove that

$$1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{6} - \frac{1}{8} + \frac{1}{3} - \frac{1}{10} - \frac{1}{12} - \frac{1}{14} - \frac{1}{16} + \cdots + \frac{1}{2n-1} - \frac{1}{8n-6} - \frac{1}{8n-4} - \frac{1}{8n-2} - \frac{1}{8n} + \cdots = 0.$$

REMINDER: There will be a 10 minute theory test on Friday March 21 covering the material in lectures 4,5,6.