MATH 255: Assignment 7

(due Wednesday, March 12)

1. (a) If $\lim_{n\to\infty} a_n = a$, show that $\sum_{n=1}^{\infty}(a_{n+1} - a_n) = a - a_1$.
   (b) Show that $\sum_{n=2}^{\infty} \log(1 - 1/n^2) = -\log 2$.

2. (a) Give an example of a convergent series $\sum a_n$ such that $\sum a_n^2$ diverges.
   (b) What happens if $\sum a_n$ is absolutely convergent?

3. (a) If (a_n) is a positive decreasing sequence and $\sum a_n$ converges, prove that $\lim_{n\to\infty} na_n = 0$.
   Hint: Use the Cauchy Criterion.
   (b) Give an example of a divergent positive series $\sum a_n$ with (a_n) decreasing and $\lim_{n\to\infty} na_n = 0$.

4. Determine the convergence or divergence of the following series:

$$\text{(a)} \quad \sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n}}, \qquad \text{(b)} \quad \sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n}.$$