MATH 255: Assignment 5

(due Friday, February 14)

1. Let $f_n(x) = x/(x+n)$ and let $a > 0$.
   (a) Show that the sequence (f_n) converges pointwise for all $x \in \mathbb{R}$.
   (b) Show that convergence is uniform on $[0, a]$ but not on $[0, \infty)$.

2. Let $f_n(x) = nx/(1 + n^2x^2)$ and let $a > 0$.
   (a) Show that the sequence (f_n) converges pointwise for all $x \in \mathbb{R}$.
   (b) Show that the convergence is uniform on $[a, \infty)$ but not on $[0, \infty)$.

3. Suppose that $g_{n+1}(x) \le g_n(x)$ for each $x \in S$ and that $g_n \to 0$ uniformly on S. Prove that

$$\sum_{n=1}^{\infty} (-1)^{n+1} g_n(x)$$

   converges uniformly on S.