

MATH 255: Assignment 5

(due Friday, February 14)

1. Let  $f_n(x) = x/(x+n)$  and let  $a > 0$ .
  - (a) Show that the sequence  $(f_n)$  converges pointwise for all  $x \in \mathbb{R}$ .
  - (b) Show that convergence is uniform on  $[0, a]$  but not on  $[0, \infty)$ .
  
2. Let  $f_n(x) = nx/(1+n^2x^2)$  and let  $a > 0$ .
  - (a) Show that the sequence  $(f_n)$  converges pointwise for all  $x \in \mathbb{R}$ .
  - (b) Show that the convergence is uniform on  $[a, \infty)$  but not on  $[0, \infty)$ .
  
3. Suppose that  $g_{n+1}(x) \leq g_n(x)$  for each  $x \in S$  and that  $g_n \rightarrow 0$  uniformly on  $S$ . Prove that

$$\sum_{n=1}^{\infty} (-1)^{n+1} g_n(x)$$

converges uniformly on  $S$ .