MATH 255: Assignment 3

(due Monday, February 3)

1. If $f \in \mathcal{R}(a,b)$ and g is a function on $[a,b]$ such that $g(x) = f(x)$, except possibly for a finite set of points, show that $g \in \mathcal{R}(a,b)$ and $\int_a^b g(x)\,dx = \int_a^b f(x)\,dx$.

2. If $f \in \mathcal{R}$ on $[a,b]$, prove that

$$\lim_{n\to\infty} \frac{b-a}{n} \sum_{k=1}^{n} f\left(a + k\frac{b-a}{n}\right) = \int_a^b f(x)\,dx.$$

Deduce that

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{n^2+k^2} = \frac{\pi}{4}, \qquad \lim_{n\to\infty} \sum_{k=1}^{n} (n^2+k^2)^{-1/2} = \log(1+\sqrt{2}).$$

3. (a) If $a < b$, give an example of a function f on $[a,b]$ such that $|f| \in \mathcal{R}(a,b)$ but $f \notin \mathcal{R}(a,b)$.

   (b) Give an example of a function f on $[0,1]$ such that f' exists but $f' \notin \mathcal{R}(0,1)$.

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