MATH 255: Assignment 2

(due Friday, January 24)

1. Let $(a_n)_{n\geq 1}$ be a sequence of real numbers. For $x \geq 0$, define

$$A(x) = \sum_{n\leq x} a_n = \sum_{n=1}^{[x]} a_n,$$

where $[x]$ is the largest integer $\leq x$ and empty sums are equal to zero. If f has a continuous derivative on $[1, a]$, show that

$$\sum_{n\leq a} a_n f(n) = -\int_1^a A(x)f'(x)dx + A(a)f(a)$$

using Stieltjes integrals.

2. Show that the following integrals exist and compute their value:

   (a) $\displaystyle\int_0^5 x^2 d[x]$;

   (b) $\displaystyle\int_{-2}^3 [|x|]\, d|x|$;

   (c) $\displaystyle\int_0^6 (x^2 + [x])d|3 - x|$;

   (d) $\displaystyle\int_0^2 x\, d([x] - x)$.

3. Using integration by parts, derive the following identities:

   (a) $\displaystyle\sum_{k=1}^{n} \frac{1}{n^s} = \frac{1}{n^{s-1}} + s\int_1^n \frac{[x]}{x^{s+1}}\, dx$ if $s \neq 1$.

   (b) $\displaystyle\sum_{k=1}^{n} \frac{1}{k} = \log n - \int_1^n \frac{x - [x]}{x^2}\, dx + 1$.

(Last modified 8:30 pm January 20, 2003)