MATH 255: Assignment 1

(due Friday, January 17)

1. Let $a < c < b$ and define α on $[a, b]$ by $\alpha(x) = 0$ for $a \leq x < c$ and $\alpha(x) = 1$ for $c \leq x \leq b$. Let f be any function on $[a, b]$.
   (a) State a necessary and sufficient condition for the integrability of f with respect to α. Prove your assertion.
   (b) Do the same for strict integrability.
2. State and prove Linearity Theorem B.
3. (a) Prove that $\int_a^b d\alpha = \alpha(b) - \alpha(a)$.
   (b) If $\int_a^b f\,d\alpha = 0$ for every f that is monotonic, prove that α must be a constant on $[a, b]$.