

MATH 255: Assignment 1

(due Friday, January 17)

1. Let $a < c < b$ and define α on $[a, b]$ by $\alpha(x) = 0$ for $a \leq x < c$ and $\alpha(x) = 1$ for $c \leq x \leq b$. Let f be any function on $[a, b]$.
 - (a) State a necessary and sufficient condition for the integrability of f with respect to α . Prove your assertion.
 - (b) Do the same for strict integrability.
2. State and prove Linearity Theorem B.
3.
 - (a) Prove that $\int_a^b d\alpha = \alpha(b) - \alpha(a)$.
 - (b) If $\int_a^b f d\alpha = 0$ for every f that is monotonic, prove that α must be a constant on $[a, b]$.