MATH 255: Assignment 6 (Due March 7, 2003)

This assignment is the midterm test that was given for MATH 243

1. a) Show that if $f \in R([a,b])$ and $f(x) > 0$ for all $x \in [a,b]$, then $\int_a^b f > 0$. (Hint: argue by contradiction and consider $F(x) = \int_a^x f$. Other proofs are also possible.)

b) Is the above claim still true if $f(x) \geq 0$ for all $x \in [a,b]$. Justify your answer by proving the claim or giving an example showing that it is not true.

2. Let $\{f_n\}$ be a sequence of functions which converges uniformly to $f \equiv 0$ on a set A.

a) Prove that for every sequence $\{x_n\}$ in A which converges to a point $a \in A$, we have $f_n(x_n) \to 0$ as $n \to \infty$.

b) Is the converse of the above true,i.e., if for every sequence $\{x_n\}$ in A which converges to a point $a \in A$, we have $f_n(x_n) \to 0$ as $n \to \infty$, does it follow that f_n converges uniformly to 0. Justify your answer. (Hint: think about x^n on a suitable interval)

3. Compute the following limit.

$$\lim_{x \to 0} \frac{x - \sin x}{x^3}.$$

Justify your answer by stating which theorem you are using and verifying the assumptions of the theorem.

4. Let $\{f_n\}$ be the sequence of functions defined on the real line

$$f_n = \frac{1+x}{1+nx^2}.$$

a) Find $\lim f_n$ as $n \to \infty$.

b) Is the convergence uniform over the real line? Justify your answer.