McGill University
MATH 251: Algebra 2
Assignment 2: due Wednesday, January 28, 2004

Note: The first three problems are to be solved using the result that, for a linear operator T on a vector space V, we have

$$\mathrm{Ker}((T-a)(T-b)) = \mathrm{Ker}(T-a) + \mathrm{Ker}(T-b)$$

if a, b are distinct scalars.

1. Find a formula for the n-th term of the sequence $x = (x_0, x_1, \ldots, x_n, \ldots) \in \mathbb{R}^{\mathbb{N}}$ where $x_0 = x_1 = 1$ and $x_{n+2} = x_{n+1} + 6x_n$ for $n \geq 0$.

2. Find all functions $f \in \mathbb{R}^{\mathbb{R}}$ such that $f(0) = f'(0) = 1$ and $f'' = f' + 6f$.

3. Find the n-th power of the matrix $A = \begin{bmatrix} 0 & 1 \\ 6 & 1 \end{bmatrix}$.

4. If U, V are the subspaces of $\mathbb{F}_2^5$ defined by

$$U = \mathrm{span}\{(1,1,0,1,0), (0,1,1,0,1), (1,0,1,0,1), (1,1,0,1,1)\}$$
$$V = \mathrm{span}\{(1,1,1,0,1), (0,1,1,1,0), (1,1,1,1,1)\}$$

   Find bases for $U \cap V$ and $U + V$.

5. Construct a linear operator T on $\mathbb{R}^4$ whose kernel and image are both spanned by the set $\{(1,1,0,1), (0,1,1,1)\}$. What is T^2? Show that $1 + T$ is invertible with inverse $1 - T$.