

McGill University
MATH 251: Algebra 2
Assignment 2: due Wednesday, January 28, 2004

Note: The first three problems are to be solved using the result that, for a linear operator T on a vector space V , we have

$$\text{Ker}((T - a)(T - b)) = \text{Ker}(T - a) + \text{Ker}(T - b)$$

if a, b are distinct scalars.

1. Find a formula for the n -th term of the sequence $x = (x_0, x_1, \dots, x_n, \dots) \in \mathbb{R}^{\mathbb{N}}$ where $x_0 = x_1 = 1$ and $x_{n+2} = x_{n+1} + 6x_n$ for $n \geq 0$.
2. Find all functions $f \in \mathbb{R}^{\mathbb{R}}$ such that $f(0) = f'(0) = 1$ and $f'' = f' + 6f$.
3. Find the n -th power of the matrix $A = \begin{bmatrix} 0 & 1 \\ 6 & 1 \end{bmatrix}$.
4. If U, V are the subspaces of \mathbb{F}_2^5 defined by

$$\begin{aligned} U &= \text{span}\{(1, 1, 0, 1, 0), (0, 1, 1, 0, 1), (1, 0, 1, 0, 1), (1, 1, 0, 1, 1)\} \\ V &= \text{span}\{(1, 1, 1, 0, 1), (0, 1, 1, 1, 0), (1, 1, 1, 1, 1)\} \end{aligned}$$

Find bases for $U \cap V$ and $U + V$.

5. Construct a linear operator T on \mathbb{R}^4 whose kernel and image are both spanned by the set $\{(1, 1, 0, 1), (0, 1, 1, 1)\}$. What is T^2 ? Show that $1 + T$ is invertible with inverse $1 - T$.