McGill University
Math 247B: Linear Algebra
Assignment 6: due Friday, March 31, 2000

1. Using a suitable spectral decomposition of $\mathbb{C}^2$, find the functions $x = x(t), y = y(t)$ which satisfy $x(0) = y(0) = 1$ and

$$\frac{dx}{dt} = x + y$$
$$\frac{dy}{dt} = -5x - 3y.$$

2. Using the theory of linear operators, find all solutions of the differential equation

$$y''' - 5y'' + 8y' - 4y = x^2.$$

3. Find a formula for

$$s_n = \sum_{i=0}^{n} (i+1)(2^i + 3^i).$$

Hint: Show that the sequence whose n-th term is s_n is in the kernel of $(L-1)(L-2)^2(L-3)^2$, where L is the left-shift operator.

4. Let S, T be diagonalizable linear operators on a finite-dimensional vector space V. Show that S, T are simultaneously diagonalizable, i.e., there is a basis of V consisting of vectors which are eigenvectors for both S and T, if an only if $ST = TS$. Hint: If $ST = TS$, show that any eigenspace for T is S-invariant.